Particle Transport from the Heliosphere to the Ionosphere Using Discrete Exterior Calculus

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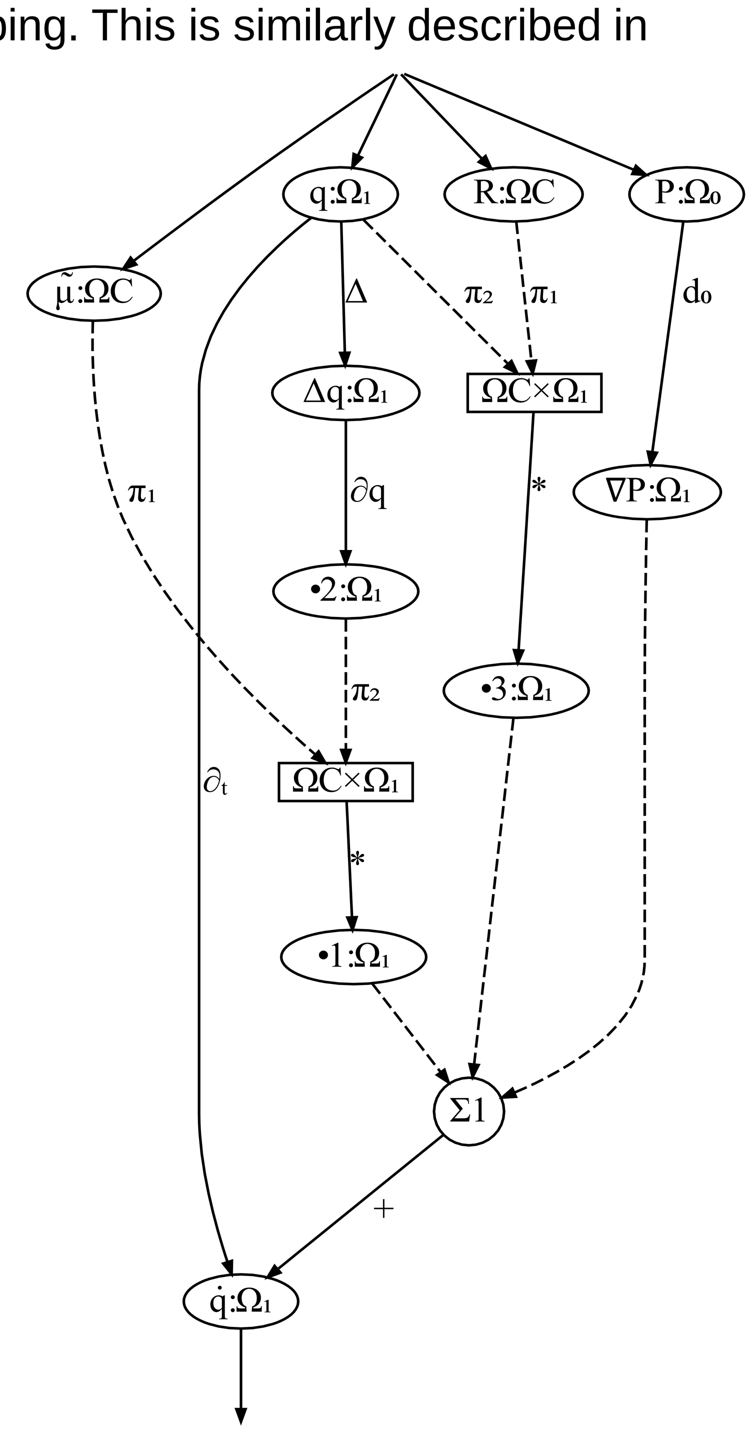
Abstract

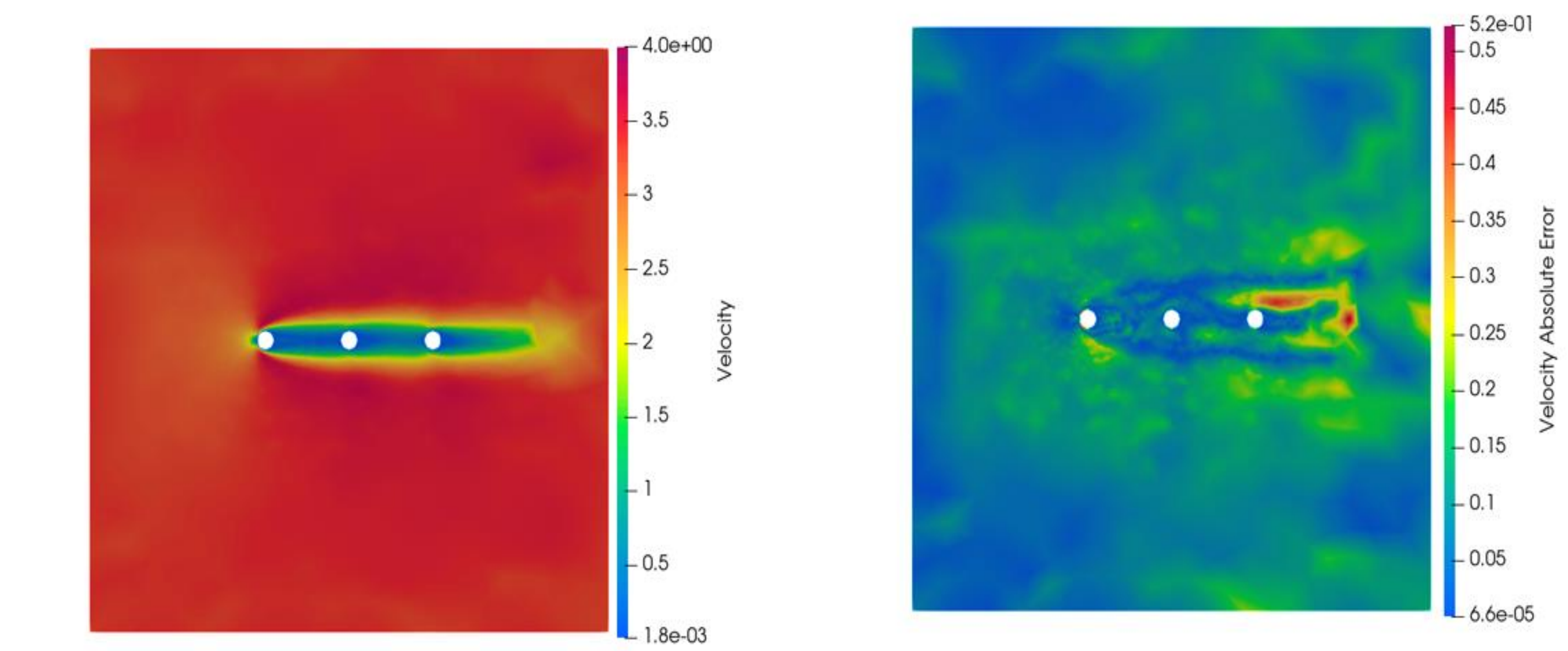
Our research aims to efficiently model the trajectory of space weather radiative particles and their impact on spacecraft throughout the environments near the Earth and the Sun. We plan to create models using a unique approach that is grounded in discrete exterior calculus. Opposed to typical models that are based within continuous calculus, this method expects to work better within applicable space environments that tend to be less continuous. Our main objectives include predicting the impact of damaging solar events for spacecraft and ground-based electronics, decreasing computational time of particle transport models, and after verification and validation of our increased capability models, analyzing the respective Sun and Earth environments and their convoluted physics.

Discrete Exterior Calculus Applied to Partial and Ordinary Diff. Eqns. (Decapodes)

Just like how continuous calculus has linear algebra, discrete exterior calculus has exterior algebra. Exterior algebra and calculus are tailored to work on discrete structures such as meshes or graphs. While linear algebra is made up of vectors that describe any unit in dimensional space, exterior algebra is made up of volumes called k-vectors that can also describe units in dimensional space. The "k" represents the dimension that the vector represents. For example, a 1-vector would denote a length, a 2-vector would denote an area, and a 3-vector is a volume; this pattern keeps going into N-dimensional space. In linear algebra, a vector has both direction and magnitude of the quantity it is describing. This is similarly described in exterior algebra.

Figure: Decapode graphical paradigm of Poiseuille flow. This paradigm shows the discrete exterior calculus interactions between processes that results in Poiseuille flow. This includes the discrete exterior derivative (d), the Hodge star operator (∗), the discrete codifferential (∂), and the discrete Laplacian (Δ). https://algebraicjulia.github.io/Decapodes.jl/dev/poiseuille/ (1)





(a) Velocity Field Computed with Decapodes.jl

(b) Absolute Error between Velocity Calculation of Decapodes.jl and SU2

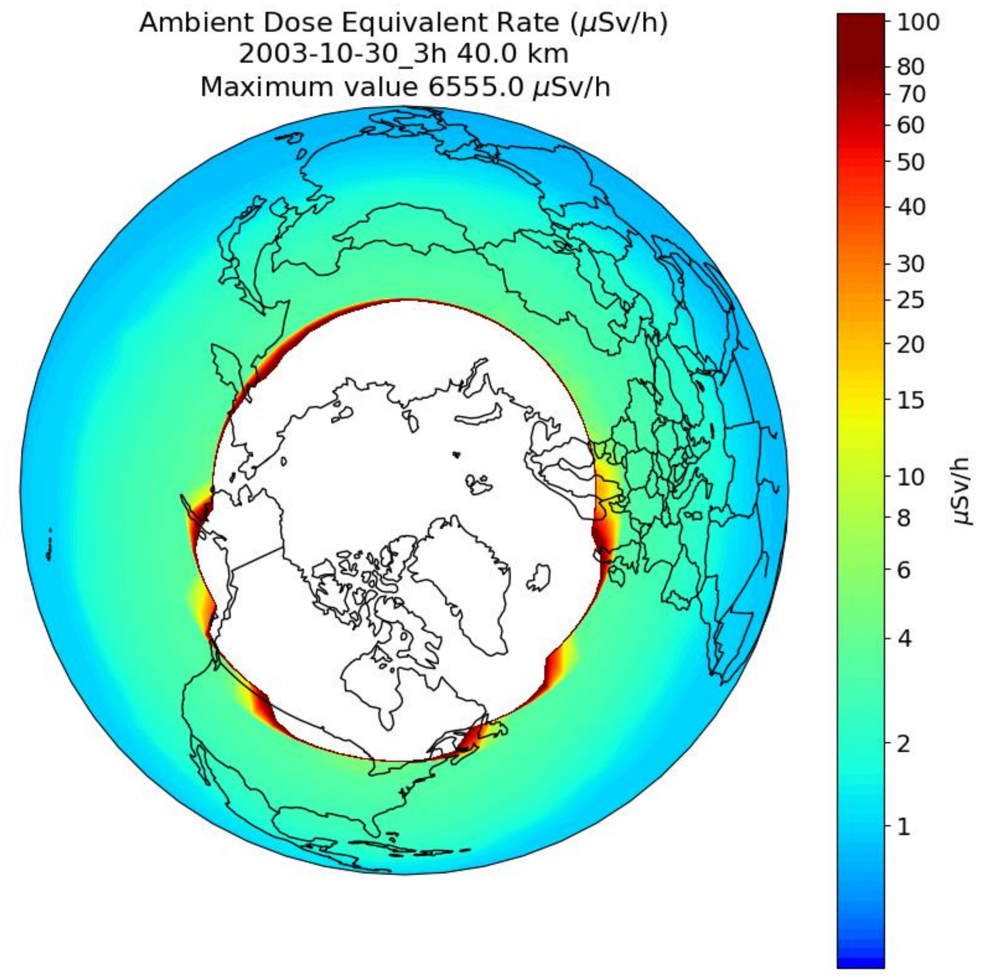
Figure: Computing the Navier-Stokes equations for fluid flow using Decapodes and comparing to state-of-the-art PDE solver SU2. (2)

Methodology Timeline

Our approach looks to verify our model test results from NAIRAS (Nowcast of Aerospace Ionizing Radiation System) using our proposed method, Decapodes. We aim to create our second unique model using Decapodes using carefully chosen transport equations for GCRs, SEPs and TRPs. We will continue to run iterations on this model to expand the capability, while also analyzing more complex environments that are suited to Decapodes' mathematical structure. With each iteration, we plan to compare our model to real-life results and observations to reaffirm, or challenge, our current perceptions of the heliospheric environment.

I will work jointly with experts within the field of Decapodes, Dr. Fairbanks and Luke Morris, to construct a model based in Decapodes. This model will be a simplified version of an existing phenomena, fluid flow through a pipe. We then will begin to work on verifying our model with NAIRAS and preparing a publication that will compare the performance of the NAIRAS model to our creation using data from the 2003 Halloween storms.

Figure: Estimated dose rate during 2003 Halloween Storms at 40km above the surface using NAIRAS . We plan on comparing our output from our model to the data retrieved from previous runs at the same time interval. (3)



Methodology Timeline Continued

We then will create a new model, with results comparable to our first created model and NAIRAS. The goal of this new model is to validate our model and results to data from the Geostationary Operational Environmental Satellite (GOES) during a new event, such as the St. Patrick's Day storm in March 2015.

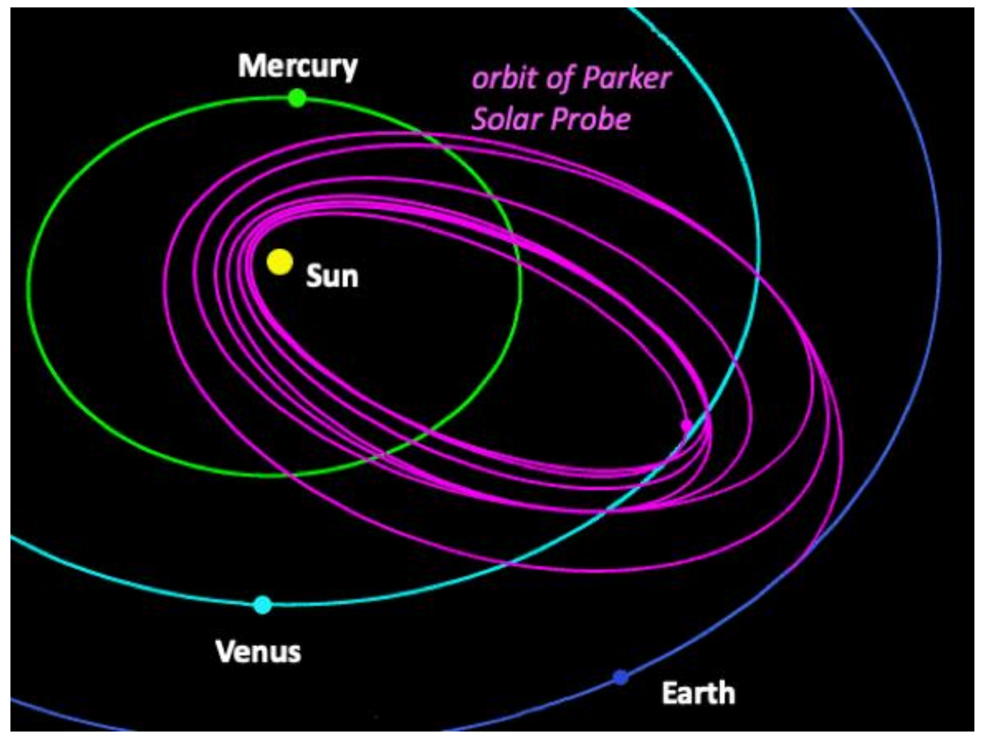


Figure: Parker Solar Probe's (PSP) orbit as it studies the Sun. We plan to compare the PSP data to our models validated by GOES and PSP to assess reasons for the differences.(4)

The final major step consists of increasing the capability and structure of the model. We can expand the capabilities to introduce new physics that are representative of intricate pieces within the space weather environment. This includes changing the transport equations for one or more of the main types of radiation transport or introducing transport equations for outer radiation belt electrons and other secondary particles from cosmic ray interaction. We will incorporate these features to compare to real-life results and observations from Parker Space Probe (PSP). Additionally, with PSP's unique orbit around the solar system, we can compare the obtained data to our model that was validated with GOES that is evaluated closer to Earth. Thus, our second model must take in data at specified times at which PSP was functioning and taking meaningful data.

Future Projects and Interests

In addition to this project, I plan on delving into the statistical modeling of space weather. I want to learn more about the statistical makeup of the space environment and research new methods to continue to improve the accuracy of space particle simulations. My topics of future interest include, but are not limited to:

- Improving Monte Carlo simulations and techniques
- Analyzing varying distribution functions of particle events
- Implementation of unsupervised machine learning into space weather

References:

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