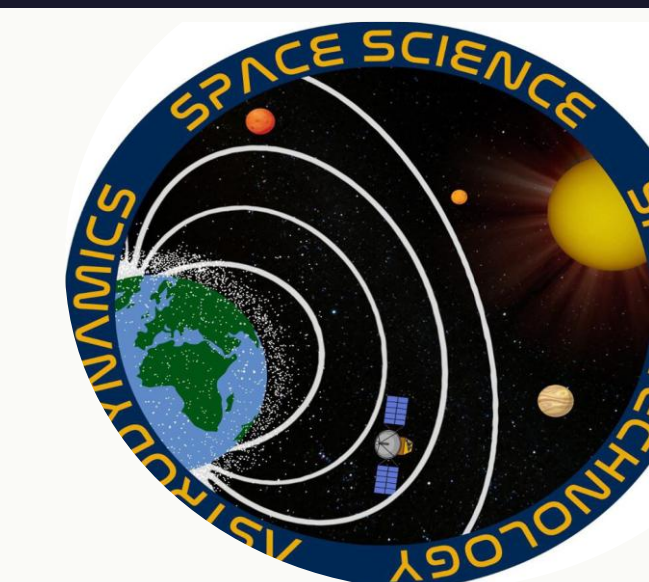


# Uncertainty-Aware Spacecraft Guidance & Navigation Under Space Weather

Nijanthan Vasudevan<sup>1</sup> Piyush M. Mehta<sup>1</sup> Mrinal Kumar<sup>2</sup>  
 West Virginia University<sup>1</sup> · The Ohio State University<sup>2</sup>



## The Problem

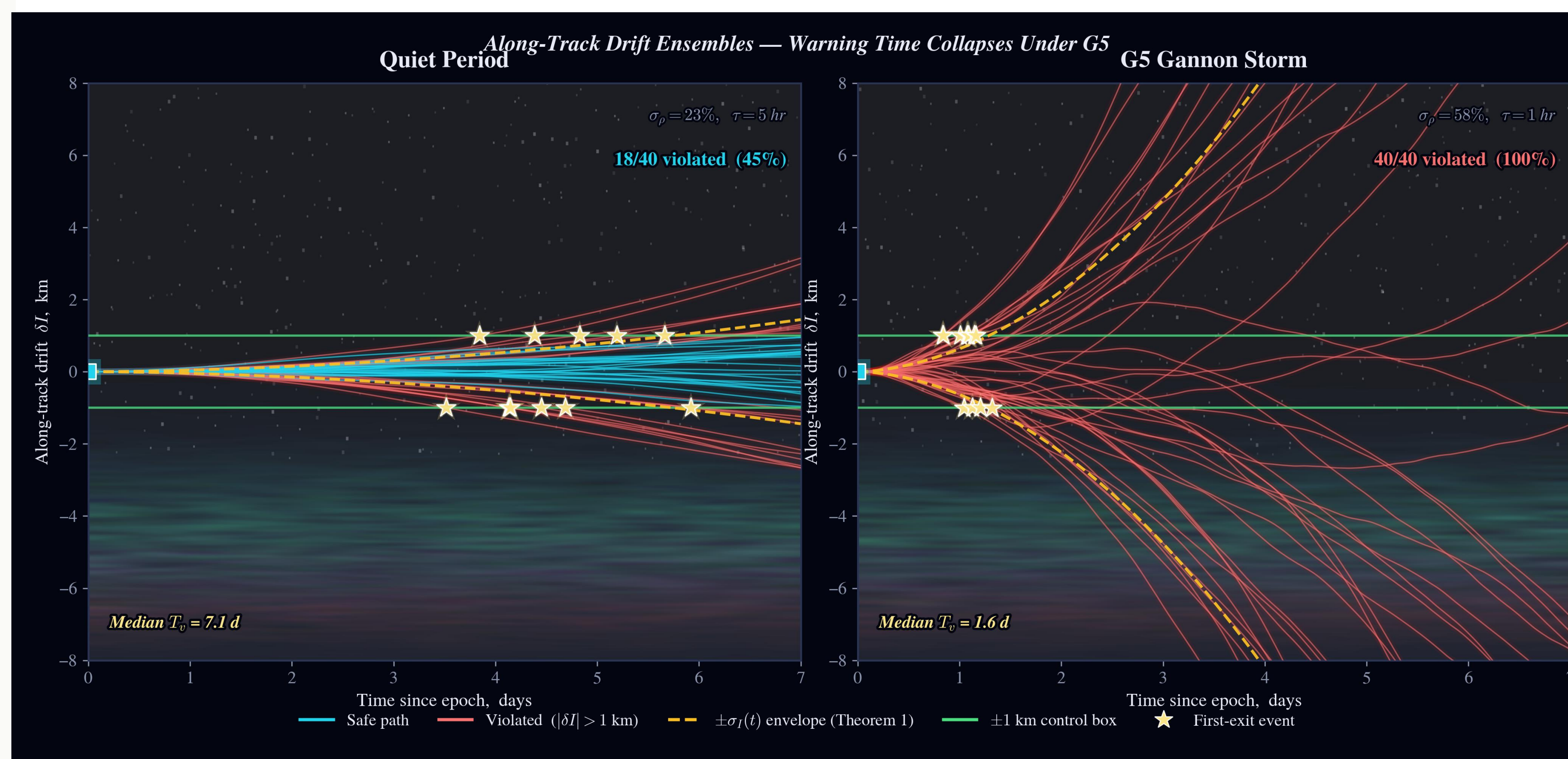
Thermospheric density is the dominant error source in LEO orbit prediction. Empirical models disagree by 20–40% during quiet conditions, and densities enhance by factors of 2–5× within hours during geomagnetic storms. Current space-weather products deliver calibrated density uncertainty  $\sigma_\rho$  — but the translation to operational decisions stops there. No closed-form expression exists in the open astrodynamics literature for the probability that a satellite violates its station-keeping tolerance by time  $t$ . Operators are left with two options: apply a blanket conservative margin or wait for post-event tracking data.



## Warning Time Collapses Under Geomagnetic Storms

Ensemble of along-track drift trajectories  $\delta I(t)$  under quiet vs. G5 Gannon storm conditions,  $\pm 1$  km station-keeping tolerance.

Each trajectory is a Monte Carlo realization of the along-track drift under calibrated density uncertainty. Green paths remain inside the  $\pm 1$  km control box; red paths violate. Stars mark first-exit events. Under G5 conditions, 100% of trajectories violate by day 2, versus <5% under quiet conditions at the same epoch.



## From Uncertainty to Decision

We derive a closed-form first-exit CDF that maps any calibrated density uncertainty forecast directly to a per-satellite maneuver-decision product

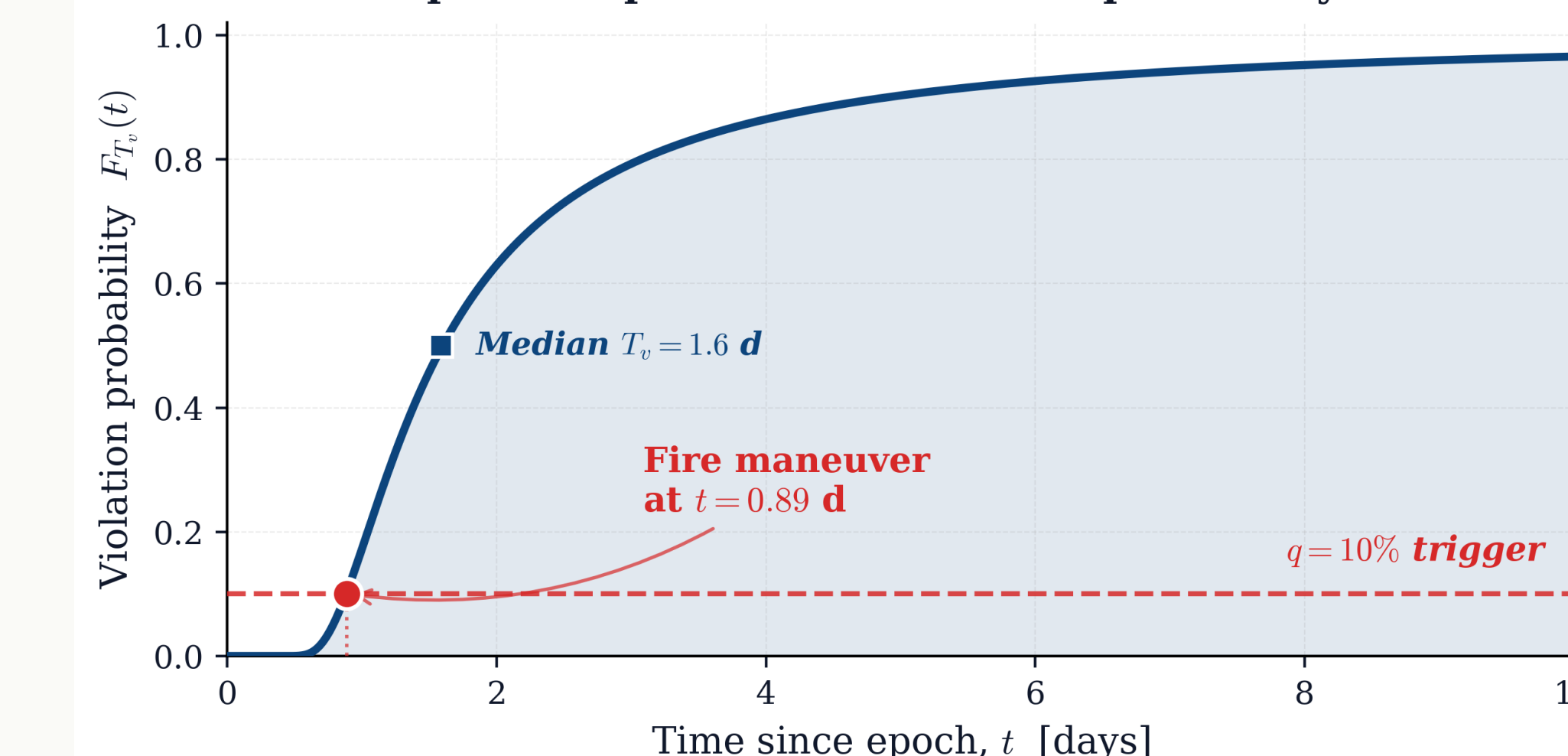
**Theorem 1** — First-exit CDF for Gaussian density uncertainty

$$F_{T_v}(t) = 2 \left[ 1 - \Phi \left( \frac{\Delta t_{max}}{\sigma_I(t)} \right) \right] + \mathcal{R}(t)$$

$$\mathcal{R}(t) < 4.7 \times 10^{-3} \text{ (operationally negligible)}$$

Along-track drift  $\delta I(t)$  is Gaussian at every epoch; first-exit probability from the Gaussian tail.

### Decision product: per-satellite violation probability vs. time



- 50  $\mu$ s per satellite (closed-form CDF eval)
- $\sim 10^4 \times$  faster than Monte Carlo (10,000 sims @  $t = 2$ )
- 4 S/C validated against TLE + GPS truth

## The Framework

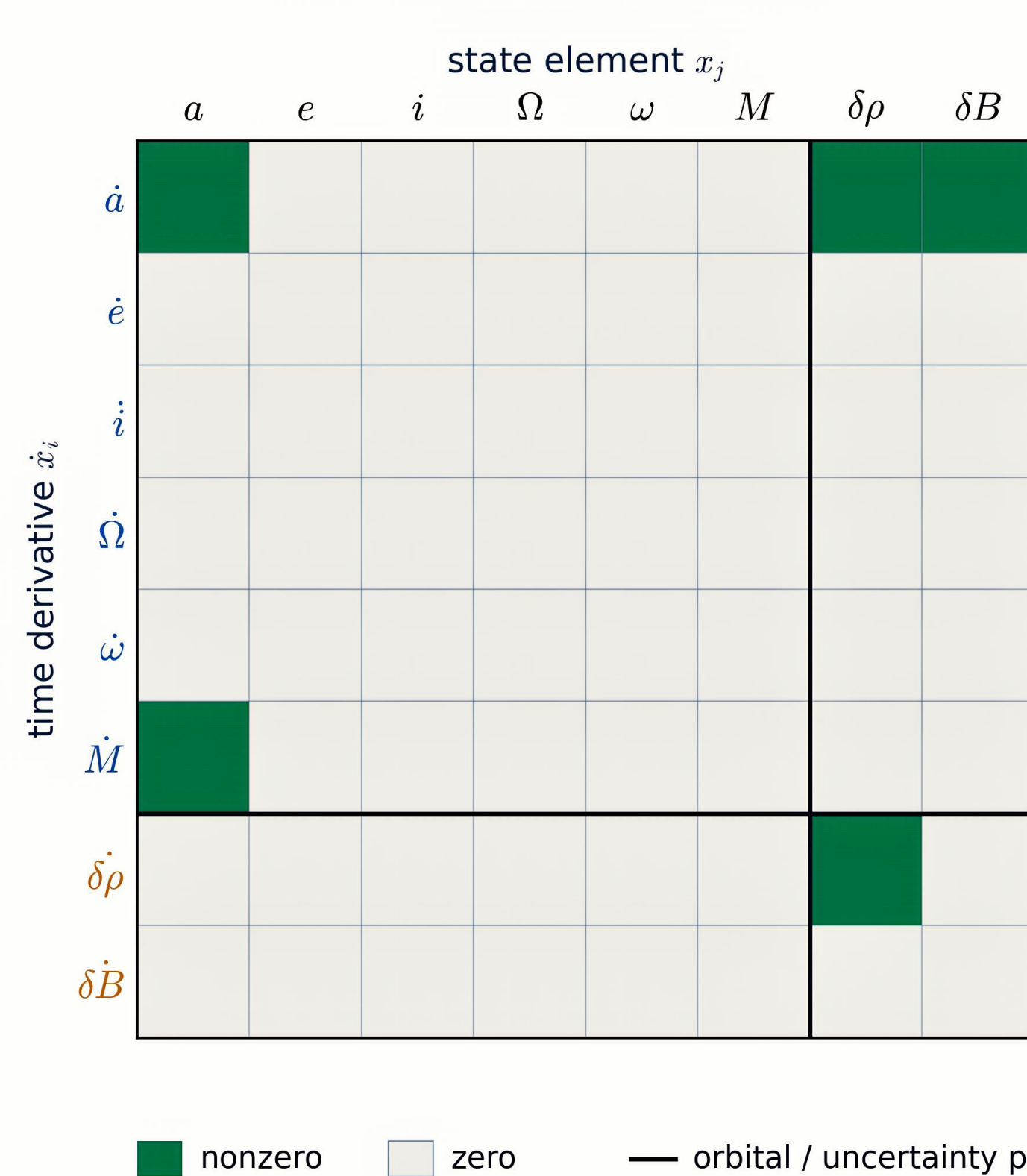
Density and ballistic coefficient uncertainty propagate through drag-specialized Gauss Variational Equations to produce along-track covariance  $\sigma I(t)$ , which feeds the first-exit CDF.

### Augmented Jacobian $F = \partial f / \partial x$

5 nonzero entries of 64 (92% sparse)

#### Augmented state

- 6 orbital:  $a$  (semi-major axis),  $e$  (eccentricity),  $i$  (inclination),  $\Omega$  (RAAN),  $\omega$  (arg. of periapsis),  $M$  (mean anomaly)
- 2 uncertainty:  $\delta \rho$  (density error),  $\delta B$  (ballistic coef.)

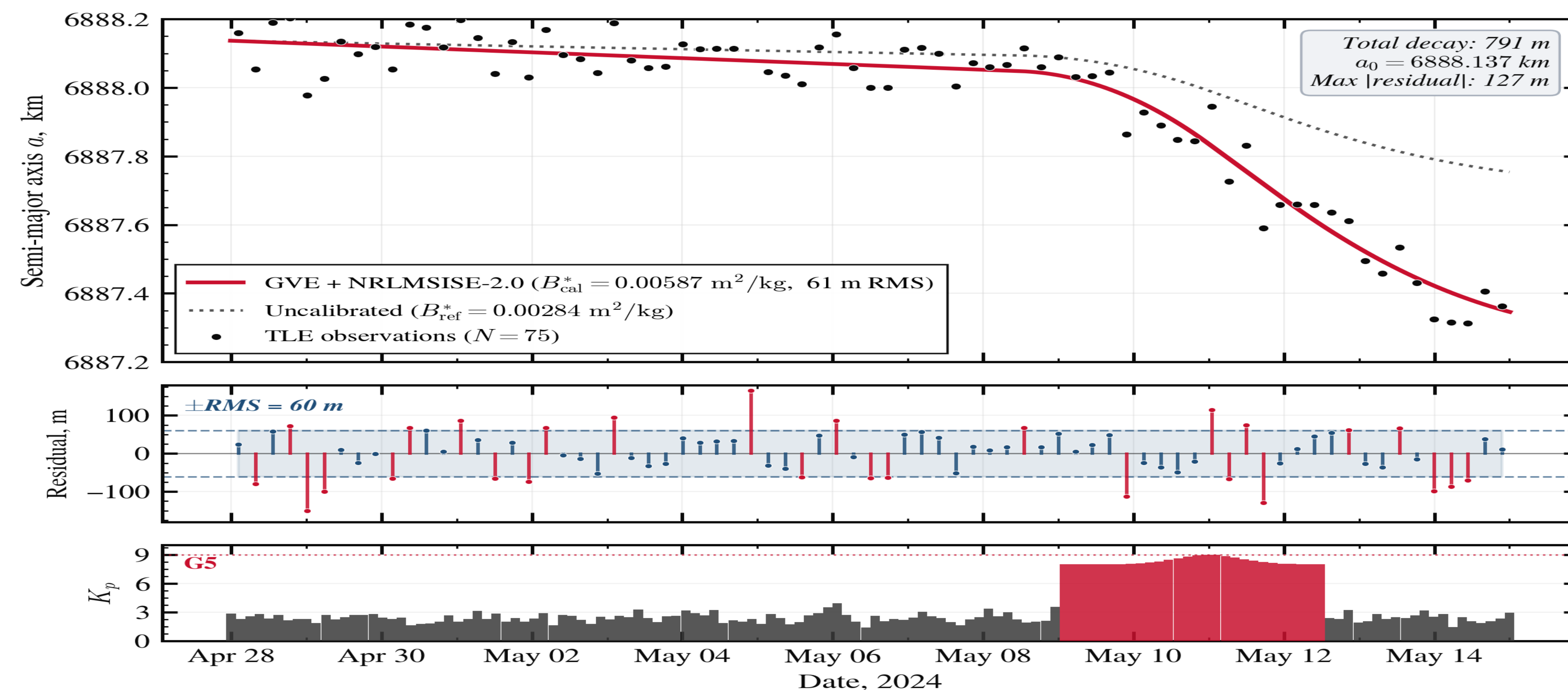


#### Density-to-drift chain

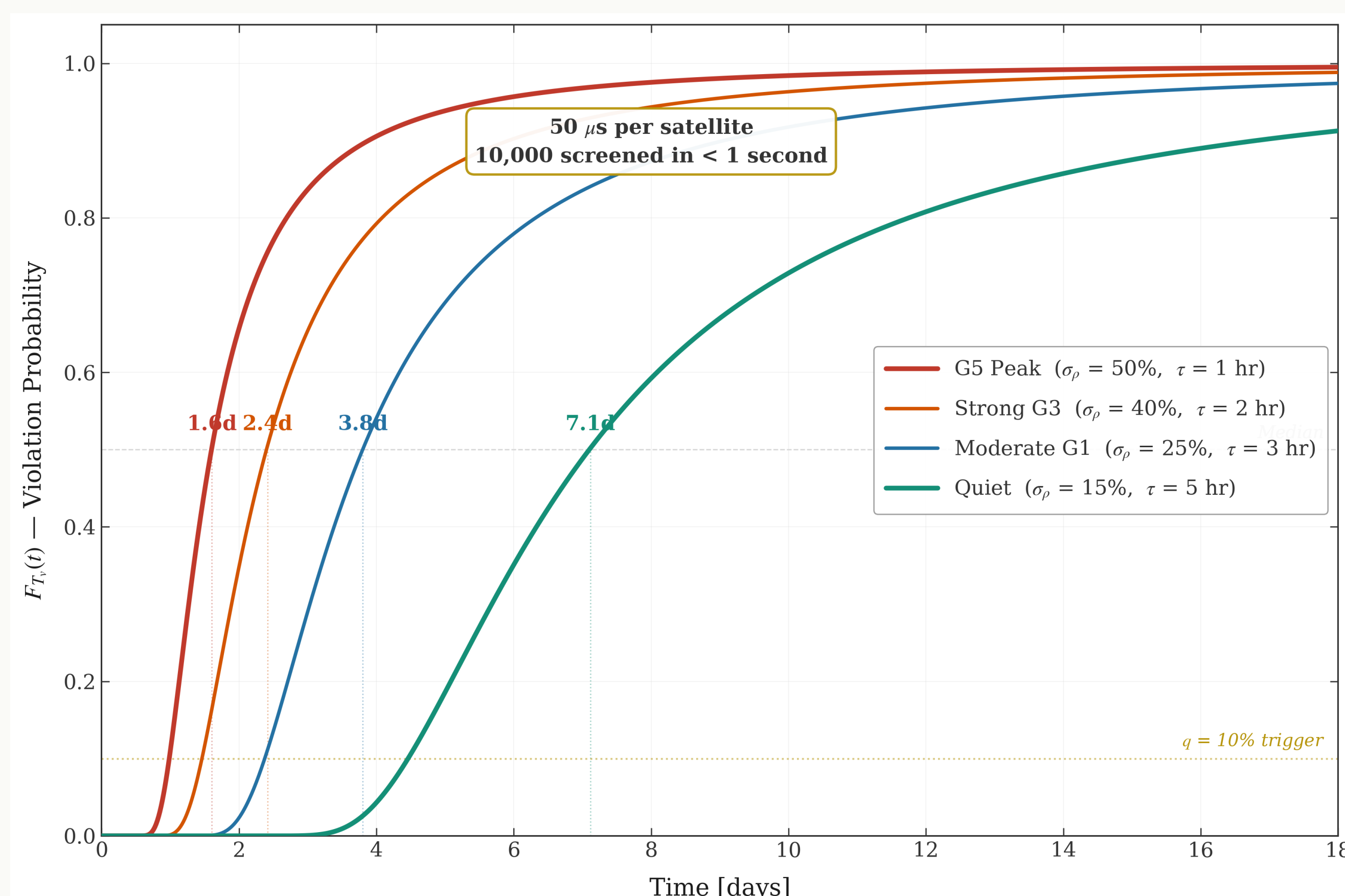
- $\delta \rho(t)$  density error (OU;  $\sigma_\rho$ , correlation  $\tau$ )
  - $\delta \dot{a}$  SMA rate perturbation ( $F_{17}: \dot{a}_{anom}(\delta \rho + \delta I)$ )
  - $\delta n$  mean-motion shift ( $F_{16}: \frac{\partial n}{\partial a} \delta a$ )
  - $\delta I(t)$  along-track drift (double integral of  $\delta \rho(t)$ )
- Linearization:  $|\sigma_\rho^{(2)} / \sigma_\rho^{(1)} - 1| \leq 5\%$  (day 3,  $\sigma_\rho \in [15, 150]\%$ )  
 K-S test:  $D = 0.013$  ( $N = 5000$ , fail to reject Gaussianity)

## Real Orbit Fit

GVE propagator validated against KANOPUS-V 3 TLE data during May 2024 Gannon G5. Calibrated  $B^* = 0.00587 \text{ m}^2/\text{kg}$  over 75 observations. 61 m RMS over 791 m of observed decay (7.7%).



## First-exit CDF: theorem 1 across storm conditions

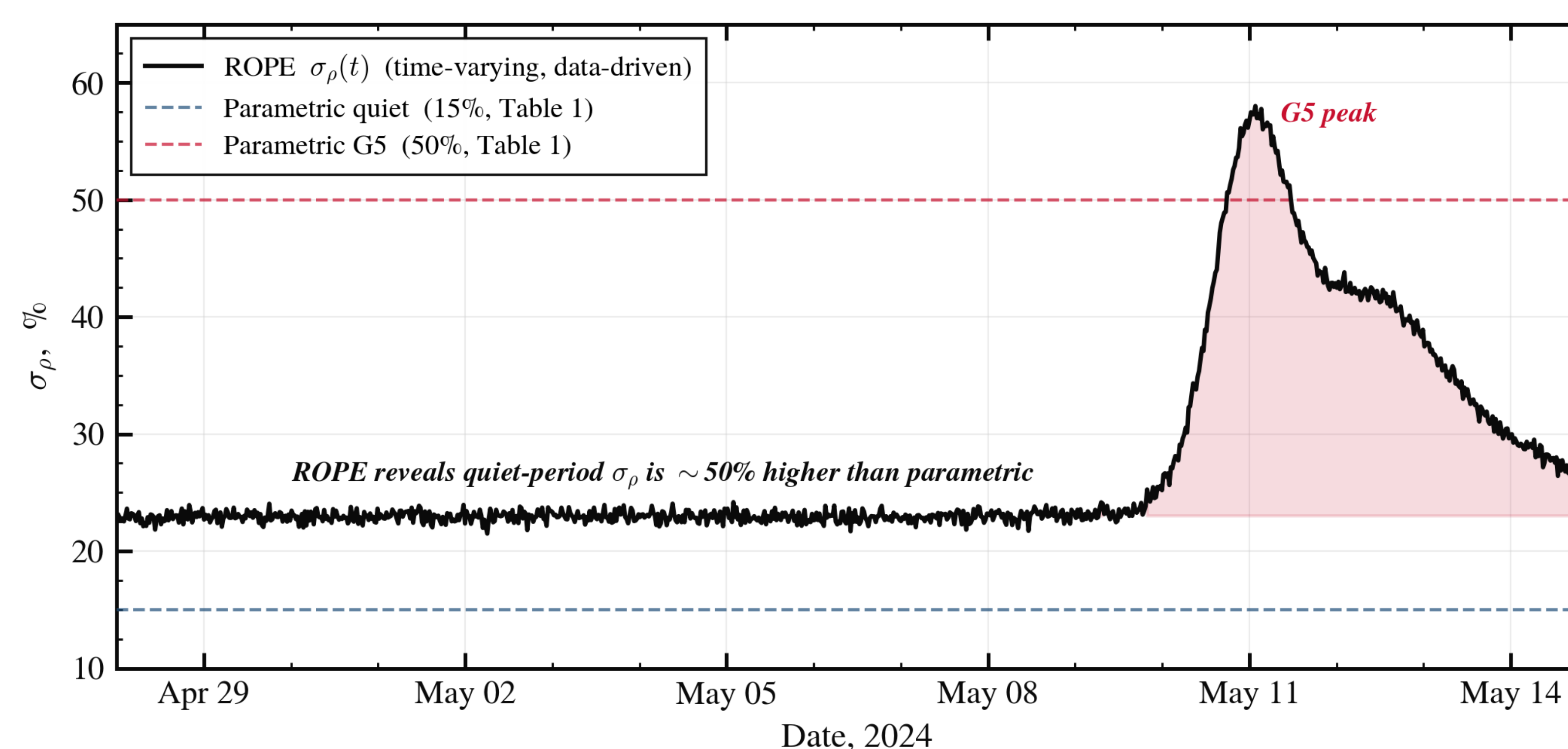


Median lead time degrades from 7.1 days (quiet)  $\rightarrow$  1.6 days (G5 peak) a 4.3× reduction. Scaling laws  $T_{v,50} \propto \Delta t^{(1/2)}$   $T_{v,50} \propto \tau^{(-1/3)}$  confirmed to  $\pm 2\%$ .

## Toward ROPE Integration

Time-varying  $\sigma_\rho(t)$  from data-driven ensemble replaces static assumptions. Static parameterizations systematically underestimate quiet-time station-keeping risk.

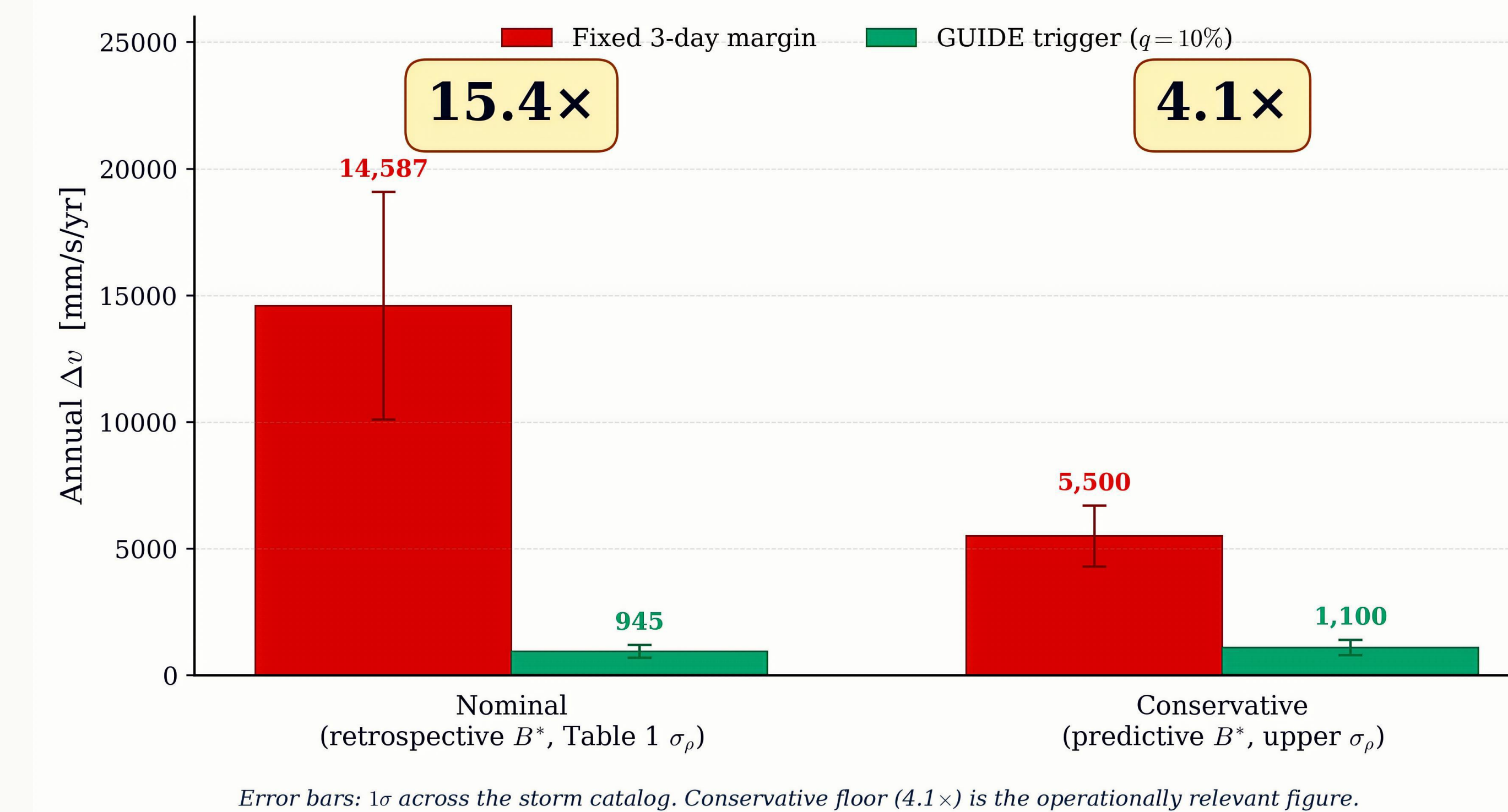
### ROPE Integration — Time-varying density uncertainty



## Operational Impact

GUIDE trigger ( $F_{T_v} > 10\%$ ) vs. fixed 3-day margin across 30 years of realistic space weather. Triggers deliver at least 4.1× fuel savings under worst-case calibration assumptions, rising to 15.4× under nominal conditions.

### 30-Year Closed-Loop Simulation — Richardson-Cane Storm Catalog



Error bars:  $1\sigma$  across the storm catalog. Conservative floor (4.1×) is the operationally relevant figure.

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## CONTACT

Nijanthan Vasudevan Ph.D. (Aerospace Engineering), ASSIST Lab, West Virginia University  
 Email: nv00003@mix.wvu.edu

