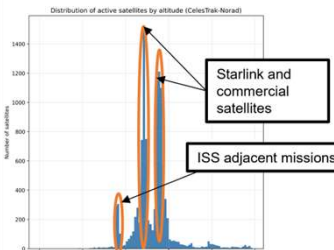


# ROPE: A Reduced-Order Probabilistic Emulator for Fast and Uncertainty-Aware Thermospheric Density Emulation

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## Why the Challenge of Space Weather



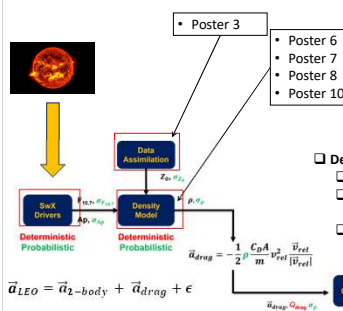
Most of the valuable assets around the Earth are in the Low Earth Orbit (LEO). Disruptive geomagnetic events can make damages. The ability to avoid orbital decay of valuable assets is determined by the ability to address the space weather challenge. We just released a tool to address this.

### Main Takeaways:

- The Low Earth Orbit region is congested.
- Increased probability of collision.
- Operations uncertainty due to thermospheric density.
- \$ value of space assets is expected to become 10x larger in the next 4 years.
- Potential billion-dollar losses during moderate disruptive space weather events (NOAA).

Monitoring debris is important: only 1% of debris that can cause mission-ending damage is currently tracked, and knowing their trajectory is of the utmost importance.

## How we are Addressing the Challenge



The largest source of uncertainty in operations is thermospheric density. The ability to avoid orbital decay of valuable assets is determined by the fidelity with which the community can estimate orbital uncertainty and the effectiveness with which it can be used on board of a satellite.

### We focus on

- Density Modeling.**
  - Improving fidelity.
  - Making the system probabilistic.
  - Making a freely available tool.
- Data assimilation** is actively used with density modeling.
- Orbit Determination and Propagation.** Directly uses density modeling.
- All frameworks shift from a deterministic to an **Uncertainty Aware** paradigm.

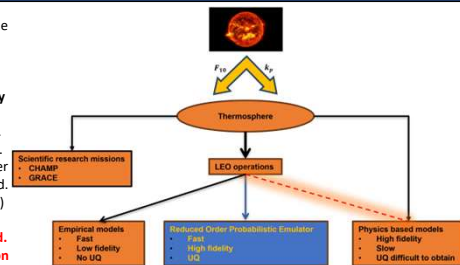
$$\bar{a}_{LEO} = \bar{a}_{z-bod} + \bar{a}_{drag} + \epsilon$$

$$\bar{a}_{drag} = -\frac{1}{2} \frac{C_D A}{m} \rho_{therm} \frac{v_{rel}}{|v_{rel}|}$$

## Operations and the Universe of Space Situational Awareness

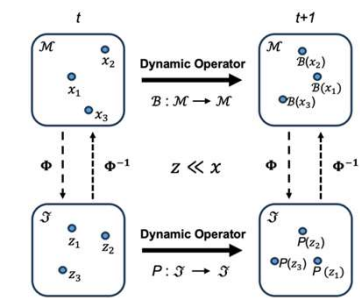
How the community simulates the thermosphere:

1. **Physics based models** (TIE-GCM, WAM) as community solution, but **computationally challenging** → ROPE.
2. **Empirical models** (NRL-MSIS).
  1. Faster than Physics based.
  2. UQ can be done, but lower fidelity than Physics Based.
3. **Data Driven Emulators (ROPE)**
  1. **High Fidelity + 10,000 X Faster than Physics Based.**
  2. **Uncertainty quantification**
  3. **Data Assimilation.**



This scheme highlights how the Sun-Thermosphere-Earth-Operations environment interconnects. The upper atmosphere expands and compresses nonlinearly due to the Sun's activity. The solar drivers  $F_{10.7}$  and  $K_p$  are proxies for this behavior and  $K_p$  is a key driver in the nonlinear evolution of the thermosphere.

## The ROPE Emulator

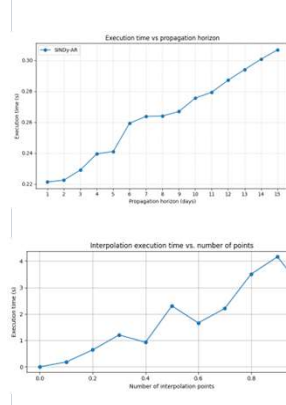


We developed our fast, high-fidelity and uncertainty aware framework to give operators an important tool to address uncertainty in operations. We use a Reduced Order Modeling Framework that uses three steps

- Dimensionality reduction.**
  - PCA exploits linearity
  - Autoencoders (CAE and COAE)
- Density Modeling.**
  - Neural Networks (NN).
  - Nonlinear regression models (SINDY(c)-AR).
- Reconstruction, interpolation, and ensemble modeling or Uncertainty Quantification.**
  - CAE and COAE are more accurate during storm-time because of the intrinsic nonlinearity of the method.
    - Just before the storm, both methods show contained errors.
    - As the storm surges, autoencoders behave much better already
    - After the storm, reconstruction errors are better controlled again with autoencoders.
  - PCA, despite being linear, is faster than autoencoders during reconstruction.

- Density modeling with NN.**
  - Generally, of **higher fidelity** due to the large number of degrees of freedom that they can leverage to emulate the thermosphere.
- Density modeling with nonlinear regression models (SINDY(c)-AR).**
  - Of high-fidelity during most part of the propagation w.r.t. the Physics Based Model.
  - With interpretable analytical structure.
  - Required work on **stability study** and constraints.

## Runtime and Calibration Analysis

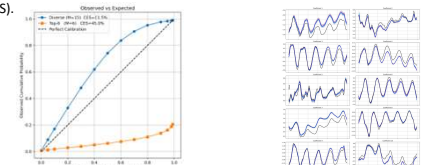


### Runtime analysis

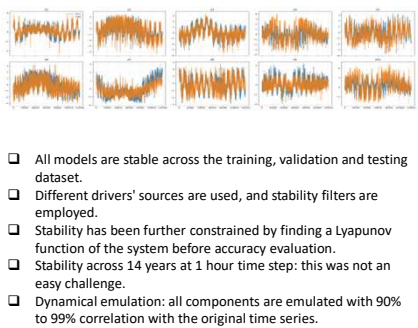
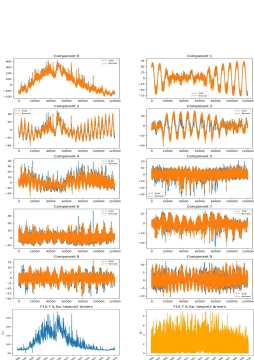
- 10,000 x faster** than physics-based models.
- Fast interpolation: constellation of 1 million satellites interpolated in 3 seconds.
  - With pre-computed matrices, it can be also **1000 times faster** than full grid interpolation.

### Calibration analysis

- Picking the best models is not always convenient.
  - Some models tend to overestimate and others to underestimate the actual coefficients.
- We pick a variety of models such that coefficients are **unbiased**.
- This diverse selection also significantly improves **Calibration Error Score (CES)**.



## Dynamic Emulation and Stability



- All models are stable across the training, validation and testing dataset.
- Different drivers' sources are used, and stability filters are employed.
- Stability has been further constrained by finding a Lyapunov function of the system before accuracy evaluation.
- Stability across 14 years at 1 hour time step: this was not an easy challenge.
- Dynamical emulation: all components are emulated with 90% to 99% correlation with the original time series.

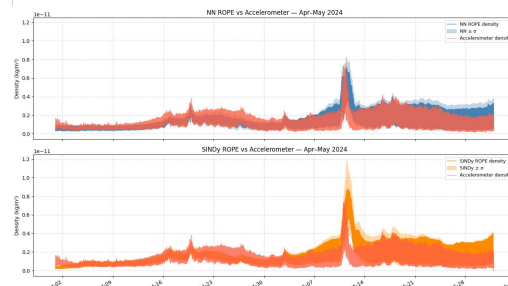
## General Conclusions

### Performance

- <5% MAPE across 14 years for storm tuned meta-models.
- 6-7% MAPE across 14 years for interpretable models.
- Fast propagation.
- High fidelity is retained across all the propagation.
- Storm tuned and meta models are the models that retain most of fidelity with TIE-GCM simulations.
- Data Assimilation is helping to tune the parameters of physics-based models and increase accuracy.
- The ROPE framework was released earlier today and is freely available. Check the QR code to start using it.**



## Testing along the GRACE-FO Orbit During a Storm



Gannon Storm: testing ROPE along an orbit.

- Ensemble model is engineered such that uncertainty increases during the storm and decreases immediately after.
- The plot shows GRACE-FO accelerometer density against ROPE outputs.
- Plot of interpretable and non-interpretable models.

- ROPE is drawn against the accelerometer derived density of GRACE-FO
- This indicates that the dynamical emulation is of high fidelity not only with respect to the Physics Based model, but also to the accelerometer density.

