Synchronizing Simulations of Turbulence to Data

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Introduction

- Magnetized turbulence is a fundamental problem that is important in understanding and predicting the evolution of our space environment. The magnetized turbulence is chaotic, hence a challenge to model and predict due to the 'butterfly effect'.
- Given that observational data are becoming more accessible, we will borrow the idea of data assimilation (DA) from numerical weather forecasting. We plan to investigate the efficacy of the most advanced DA methods in the simulations of magnetized turbulence.
- PUNCH will probe scales of turbulence within the upper end of the inertial range (large scales), which could be used to synthesize PUNCH data and numerical simulation to improve prediction.

Turbulence Modelling Approaches

- Different turbulence modelling approaches have been developed, and the three most common ones are: Reynolds-Averaged Navier-Stokes (RANS), Large Eddy Simulation (LES), and Direct Numerical Simulation (DNS).
- DNS can resolve down to small scales.

  LES solves the large eddies and models the small eddies.

  RANS simulations resolve the mean flow and average out the turbulent fluctuations.
- DNS: $\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\frac{\nabla p}{\rho} + \nu\nabla^2\boldsymbol{u}$
- LES: $\partial_t \tilde{\boldsymbol{u}} + \tilde{\boldsymbol{u}}\cdot\nabla\tilde{\boldsymbol{u}} = -\frac{\nabla \tilde{p}}{\rho} + \nu\nabla^2\tilde{\boldsymbol{u}} - \nabla\cdot\boldsymbol{\tau}^l$

  $\tau_{ij}^l = \widetilde{u_i u_j} - \tilde{u}_i\tilde{u}_j$

  subgrid-scale (sgs) stress models
- RANS: $\partial_t\langle\boldsymbol{u}\rangle + \langle\boldsymbol{u}\rangle\cdot\nabla\langle\boldsymbol{u}\rangle = -\frac{\nabla\langle p\rangle}{\rho} + \nu\nabla^2\langle\boldsymbol{u}\rangle - \nabla\cdot\boldsymbol{\tau}$

  $\boldsymbol{u} = \langle\boldsymbol{u}\rangle + \boldsymbol{u}'$

  $\tau_{ij} = \langle u_i u_j\rangle$

  Reynolds stress models

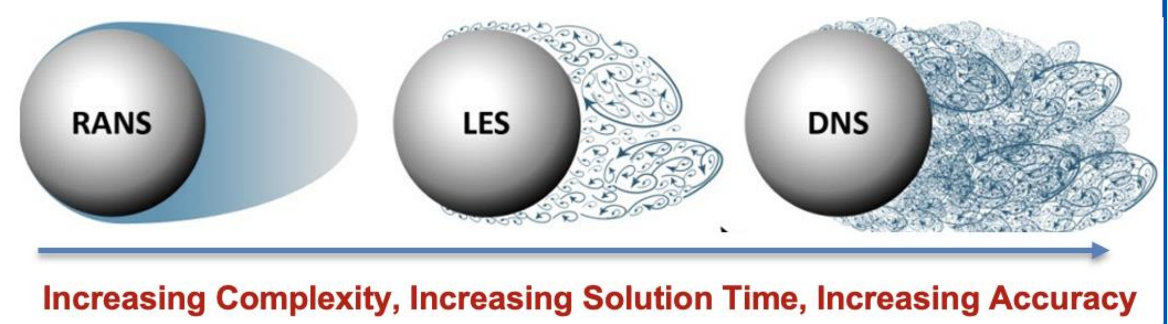

Fig.1. Example of turbulent spatial scales over a sphere resolved by modeling approach (*John Hart, Procedia engineering, 147:68–73, 2016*).

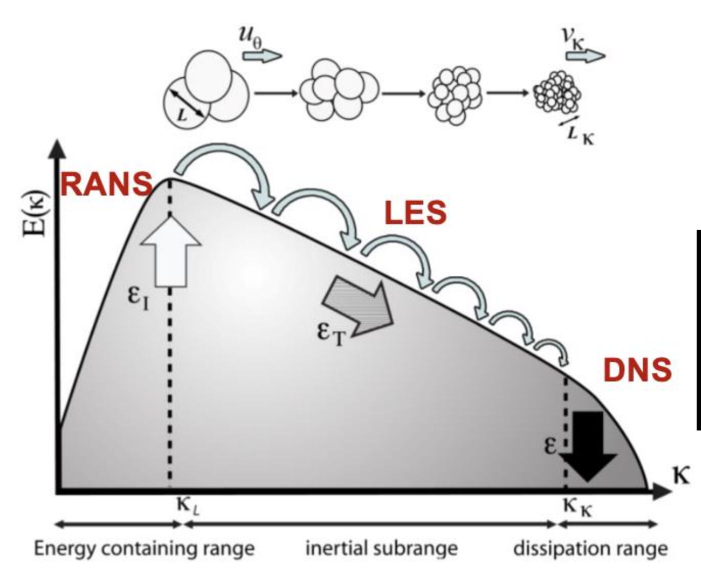

Fig.2. Sketch of the range of scales in energy spectrum.

Data Assimilation

- Data assimilation (DA) refers to a suite of techniques by which experimental data are combined with numerical simulations to improve the prediction of the numerical model.
- DA methods: Four-dimensional variational (4DVAR) method, ensemble Kalman filter (EnKF), direct substitution (DS), etc.

Synchronization of LES to DNS Using DS

- When the amount of assimilated data exceeds a threshold wavenumber, LES is synchronized with DNS.

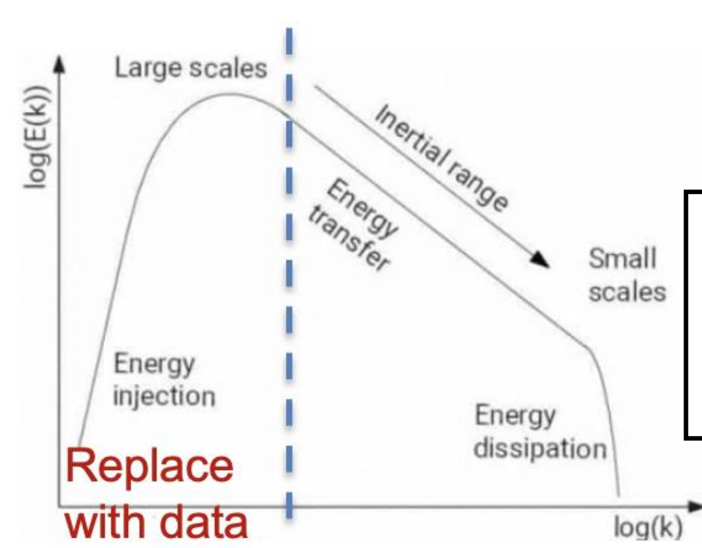

Fig.3. Energy spectrum showing large-scale fields are replaced by data.

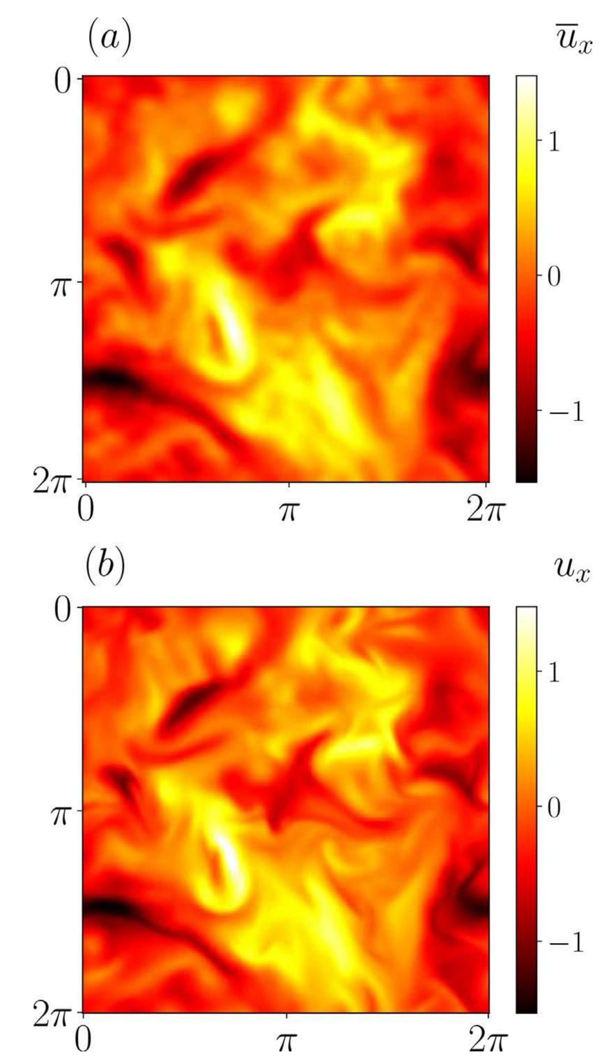

Fig.4. Instantaneous velocity distribution from (a) LES and (b) DNS (*Li et al., Phys. Fluids 34, 065108, 2022*).

Small-Scale Reconstruction of Turbulence Using 4DVAR

$$J = \frac{1}{2}\int_0^T \langle \mathcal{F}\boldsymbol{u} - \mathcal{F}\boldsymbol{v},\, \mathcal{F}\boldsymbol{u} - \mathcal{F}\boldsymbol{v}\rangle\, \mathrm{d}t$$

$$-\partial_t\boldsymbol{u} - \boldsymbol{u}\cdot\nabla\boldsymbol{u} - \nabla p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f} = 0,$$

$$\nabla\cdot\boldsymbol{u} = 0, \quad \boldsymbol{u}(\boldsymbol{x}, 0) = \boldsymbol{\varphi}(\boldsymbol{x})$$

The objective of the optimization problem is to find φ(x) such that J is minimized while u(x, t) satisfies the constraints.

- Reconstruction is successful when the resolution of the measurement data, is of the order of the threshold value $k_c = 0.2\eta_K^{-1}$ (η_K is the Kolmogorov length scale).
- Satisfactory reconstruction of the scales two or more octaves smaller is possible if data at large scales are available for at least one large eddy turnover time.

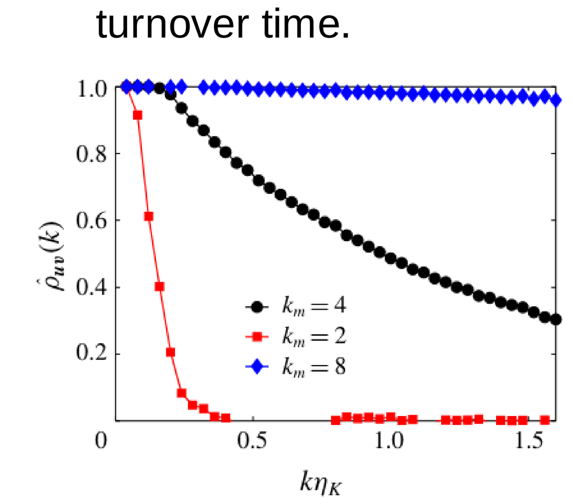

Fig.5. Correlation between target and reconstructed fields for different thresholds (*Li et al., J. Fluid Mech. 885, A9, 2020*).

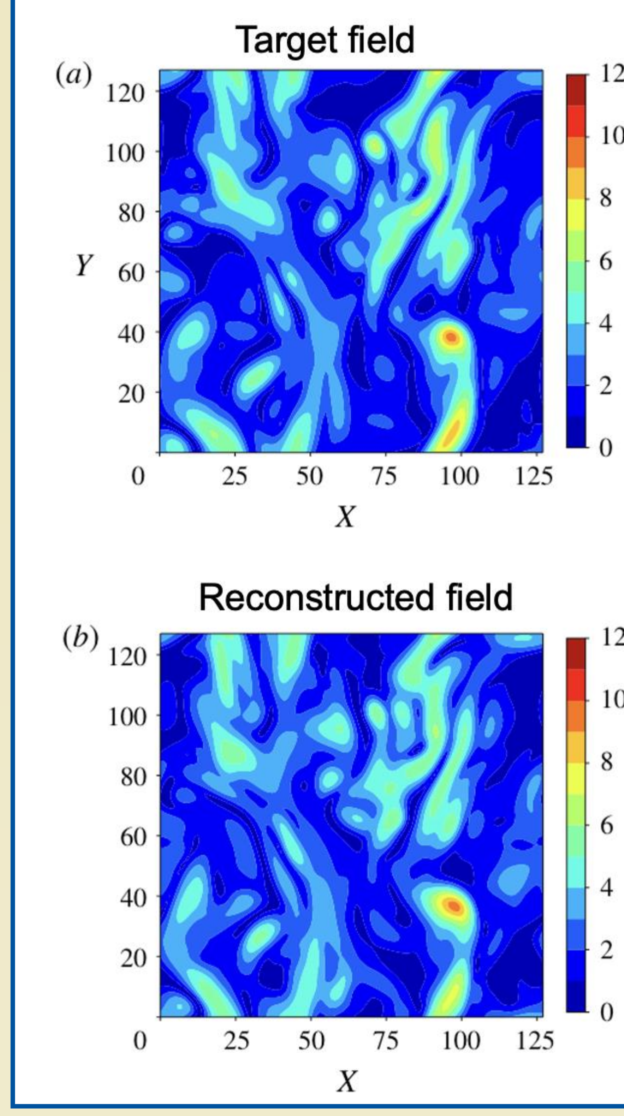

Fig.6. Distributions of vorticity on a plane. (a) from a target field; (b) from the reconstructed field by 4DVAR (*Li et al., J. Fluid Mech. 885, A9, 2020*).

Directions for Future Research

- The extreme parameter regimes encountered in heliospheric applications preclude DNS. Most of 3D magnetohydrodynamics (MHD) heliosphere models from the solar corona to the solar wind can only capture large-scale feature, which behave like RANS model. One exception is the model developed by Usmanov, which incorporates turbulence transport for global solar wind simulations.
- The central question is **how to reconstruct the small scales of magnetized turbulence, given experimental (numerical and observational) data at large scales**.
- The main goal is to **explore the ability of DA to reconstruct small scales of magnetized turbulence**.
  - **Objective 1**: Implement DA and compare the performance of DA methods in small-scale reconstruction of magnetized turbulence.
  - **Objective 2**: Implement DA methods with heliospheric data (such as PUNCH) and reconstruct small scales.

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