MHD turbulence simulations as a testing ground for PUNCH

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(1) Introduction

PUNCH will image macroscopic features of the inner heliosphere and also admit sufficiently high spatial resolution to probe scales of turbulence within the upper end of the inertial range, close to the integral scale. Because PUNCH is an imager, the measurements it will make relate differently to the underlying turbulent environment of the outer corona and inner heliosphere than do more familiar in-situ samples. This numerical study combines magnetohydrodynamic (MHD) simulations of turbulence together with synthesis of white-light data via the FORWARD code. We show that (i) the ``usual'' turbulence scalings are modified by the integration along the line of sight in an optically thin medium, and (ii) those scalings are still linked to the original properties of the turbulent field. This study is a first step in the process of analyzing and understanding the unprecedented information that PUNCH will provide.

(2) Simulation description

We used homogeneous isotropic weakly compressible MHD 3D turbulence simulations. The box size is $(2\pi L)^3$ discretized with 512^3 points.The integral scale is ~1.2, and the ratio of the root mean square density fluctuations to the mean is ~0.16 similar to observations of the inner heliosphere. **However, no MHD turbulence simulation is big enough to feature all the scales that PUNCH will encompass.** See box (3) to read about our stratagem.

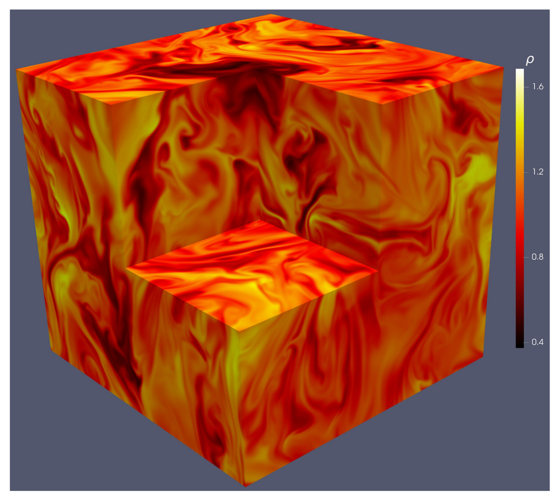

Fig. 1. 3D rendering of the turbulent density field.

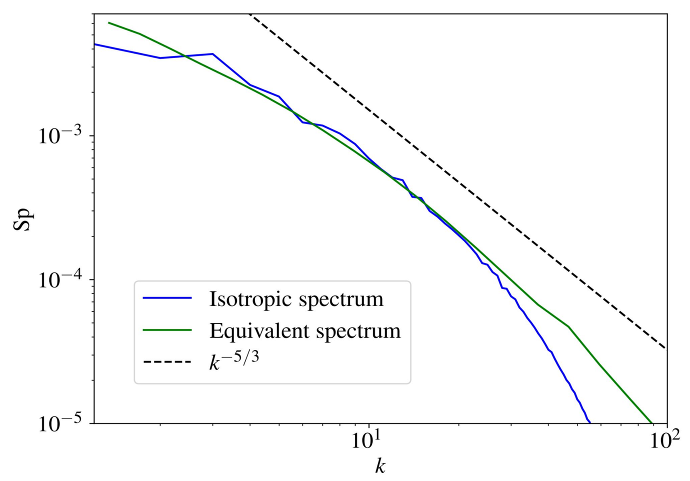

Fig. 2. Isotropic FFT spectrum and equivalent spectrum (see box (5).

(3) Rubik's cube heliosphere

We populate a cube which has sides of length 180 $R_\odot$ with multiple replicas of the simulation, tuning the simulation's spatial resolution to matche PUNCH's requirements. The "extended" density field is processed using FORWARD modeling tool (gibson2016forward) to create brightness maps similar to those provided by the mission. A radial falloff for the density to decrease as R^{-2} is included.

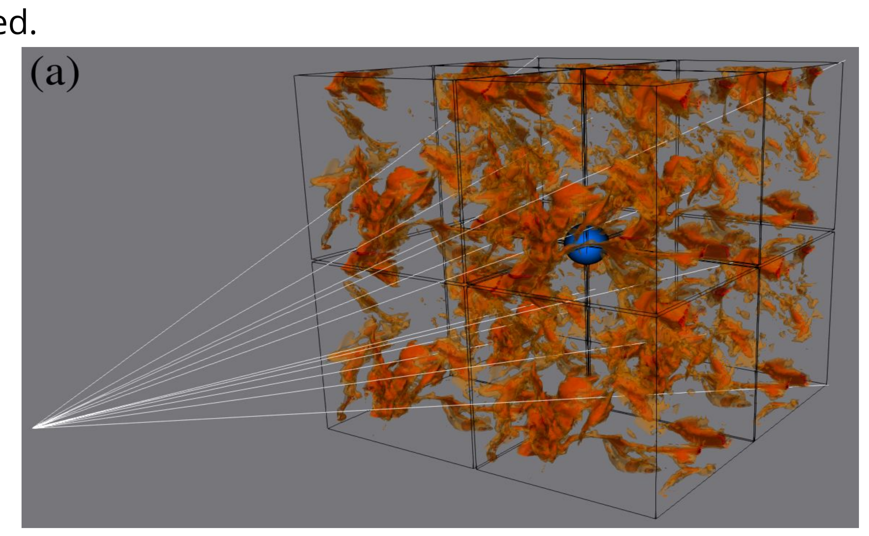

Fig. 3. Example of "Rubik's heliosphere". The Sun is the blue sphere in the center. Each cube is a replica of the cube in Fig. 1. Orange sheets represent regions of the highest density. White lines are LOS (line of sight) integration lines similar to PUNCH's.

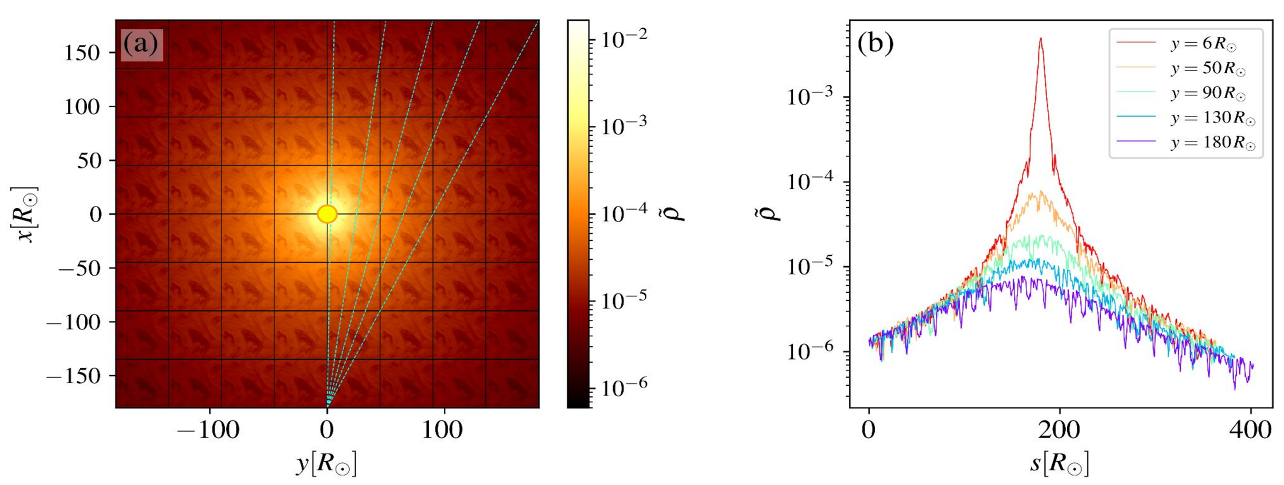

Fig. 4. One plane (z=0) of the Rubik's heliosphere. Each "tile" is a plane of the cubes in Fig.3. The sun is the yellow sphere in (0,0) (not to scale). Cyan dashed lines are analogous to the white lines in Fig. 3. The density values collected along these lines are reported in panel (b).

(4) Field of view (FOV) and integration along LOS effects

Several effects modify the FORWARD output from the initial simulations. (i) The position of the FOV produces different stretching. (ii) The radial falloff of the density field convolved with the Thomson scattering. (iii) Integration along non-parallel LOS. If we allow for all these effect (as with actual observations), we obtain a final product (9 panels to the right in Fig. 5) which features radial trends, averaged (less sharp) gradients and no periodic boundaries.

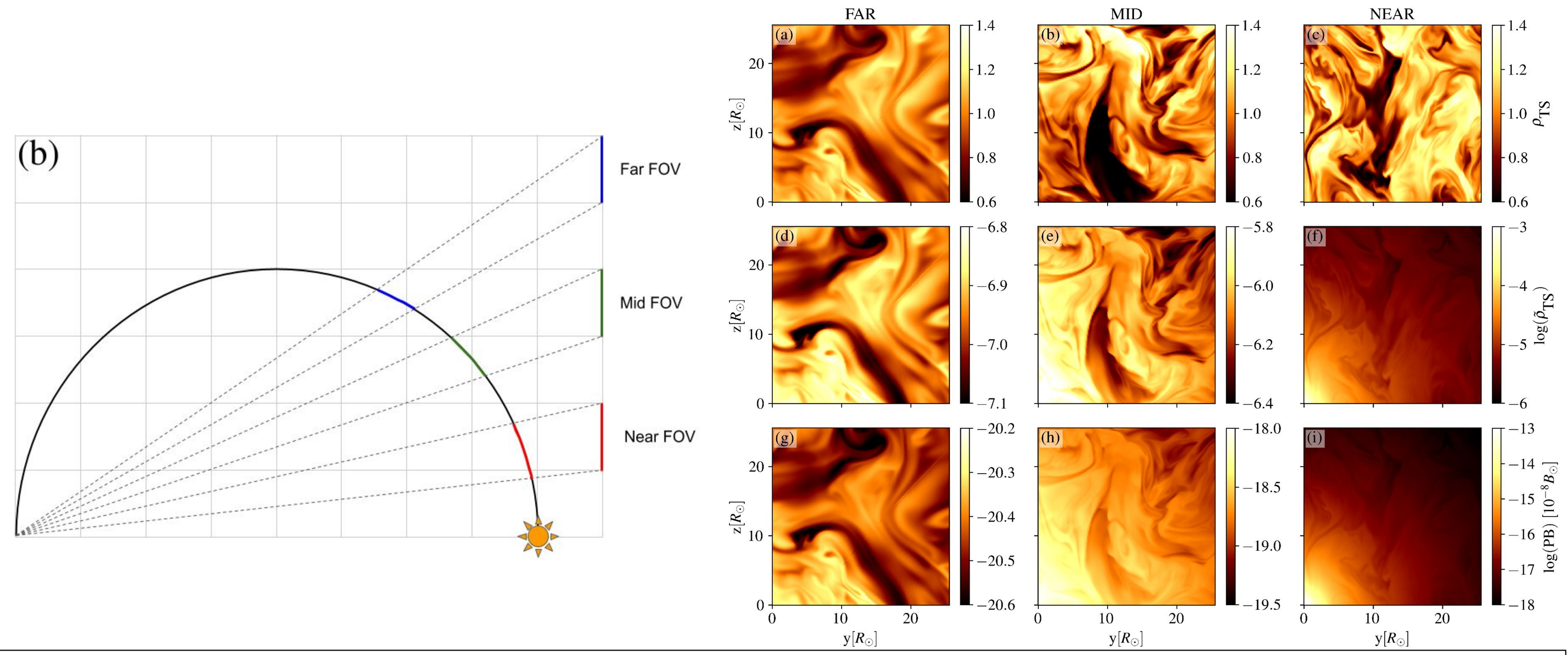

Fig. 5. (Left) sketch of the "tiled" heliosphere, the Thomson sphere (TS), and the position of three FOVs. (Right) effects of the FOV on different quantities of as they appear at the TS (no integration along LOS) . Each row shows the distortion effects due to the projection of an arc of the TS on a plane for (row 1) density, (row 2) density scaled with R^{-2}, and (row 3) polarized brightness (PB).

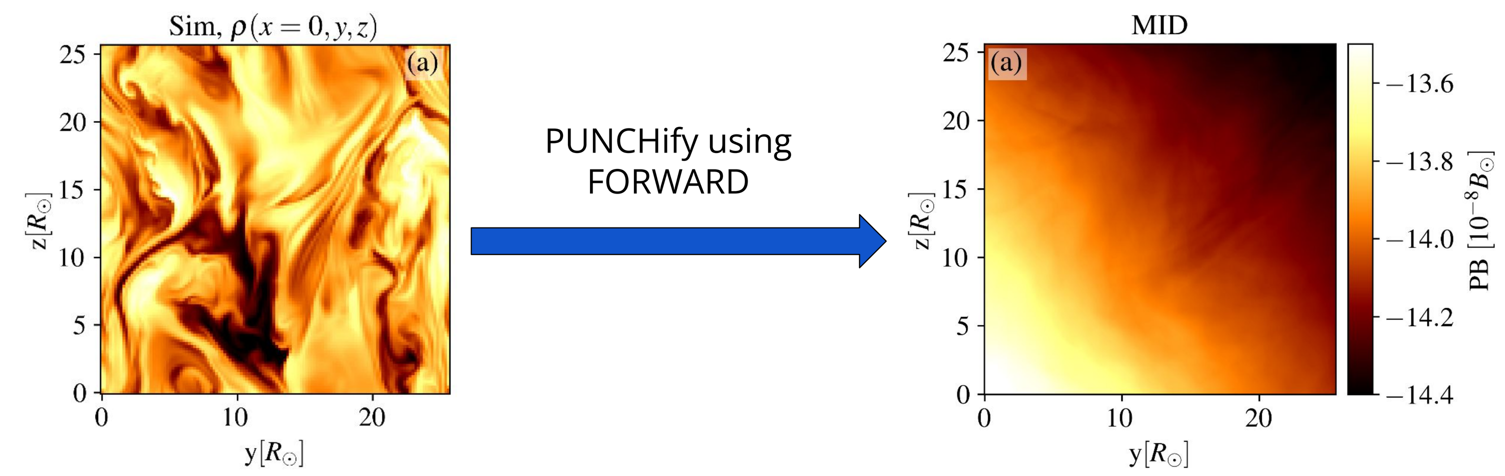

Fig. 6. Original MHD simulation of density, and the modification simulated with FORWARD for a PUNCH-like image. The effect is quite dramatic. We need to carefully treat the final product to try and obtain turbulence information.

(5) Bits of theory

One quantity of interest in turbulence theory are power spectra. Formally, the power spectrum is the Fourier transform of the correlation function that is defined, for a field *f* as,

$$R(\ell) = \langle f(x) f(x+\ell) \rangle$$

However, given the changes in the gradients and structure sizes due to LOS integration effects (Figure 6), Fourier analysis does not recover the true spectrum. Indeed, since the "PUNCHified" field is dramatically different from the original one, the field *f* in the brackets is different and therefore, its Fourier transform will provide a different result. Therefore, we need to go through some intermediate steps. Since FFTs do not retain the information we are interested in, we proceed in using the 2nd order structure function.

$$S^2(\ell) = \langle |f(x) - f(x+\ell)|^2 \rangle$$

that is directly related to the correlation function as

$$S^2(\ell) = 2E - 2R(\ell)$$

where $E = \langle f^2 \rangle$ is the total energy of the system and the bracket is an ensemble average. Therefore, it is possible to cast S^2 in the form of an "equivalent" power spectrum $P(k^*)$ as

$$P(k^*) = \ell S^2(1/\ell)$$

having defined an effective wavenumber $k^* = 1/\ell$. Fig. 2 shows the good agreement between the FFT and the equivalent spectrum..

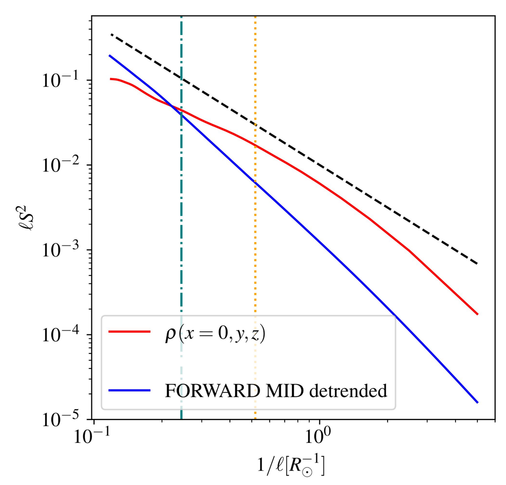

Fig. 7. Comparison of the equivalent spectra for the original field (red) and the PUNCHifed image (blu). A direct comparison of the two is evidently difficult.

(6) How do we get results to agree?

To try and extract similar turbulence properties from the two fields in Fig. 6, knowing what is in box (5), we can do the following:
(i) we average the simulation domain along one direction, instead of taking one plane only, to mimic the integration along FOV
(ii) we detrend PUNCH images to remove the radial falloff of the brightness caused by the Thomson scattering and the radial behavior of the density field.

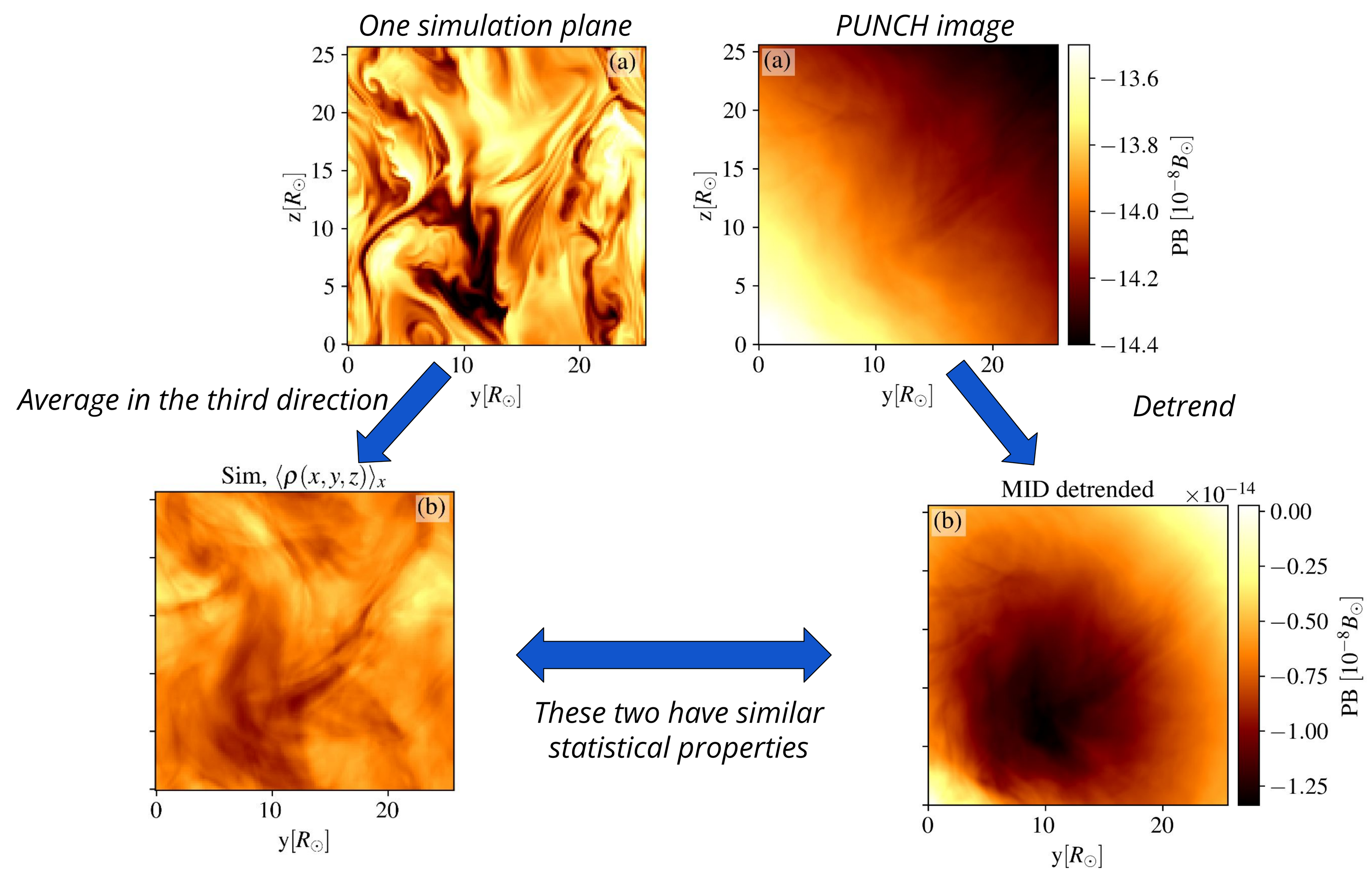

Fig. 8. Procedure to compare the statistical properties of the simulation and PUNCH fields. Even though the bottom two figures look different, their statistical properties are now similar.

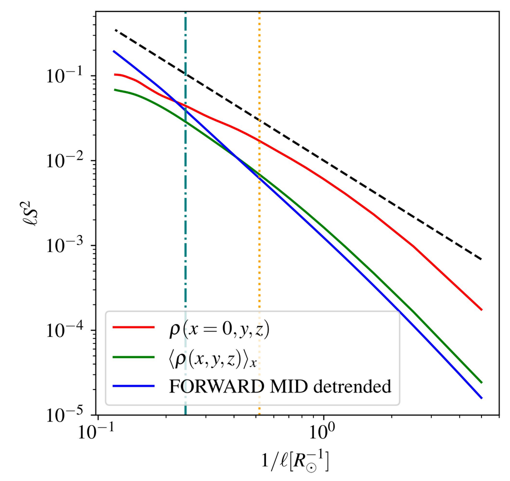

Fig. 9. Comparison of the equivalent spectra for the original simulation (red) and the modified (average) simulation (green) and PUNCH image (blue). Now the spectral properties of the latter two are similar.

(7) What we have learned and next steps

1. Spectral analyses cannot be performed directly to obtain turbulence information in the regions observed by PUNCH.
2. Data can be processed to obtain simulations and observations to agree. Even though the observed properties are different from the "pristine" field.
3. We need a database of different simulations and scenarios to map back the modified spectral features to known turbulent fields.
4. Future works will go in this direction of PUNCHify different turbulence models (slab+2D, anisotropic, compressive, ...) and find links between them.
5. This will provide unique ways to obtain information about the turbulent environment especially in regions that PUNCH will observe but that are inaccessible to spacecraft to support imaging with in situ observations.

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