$1/f$ Noise in the Heliosphere: A Target for PUNCH Science

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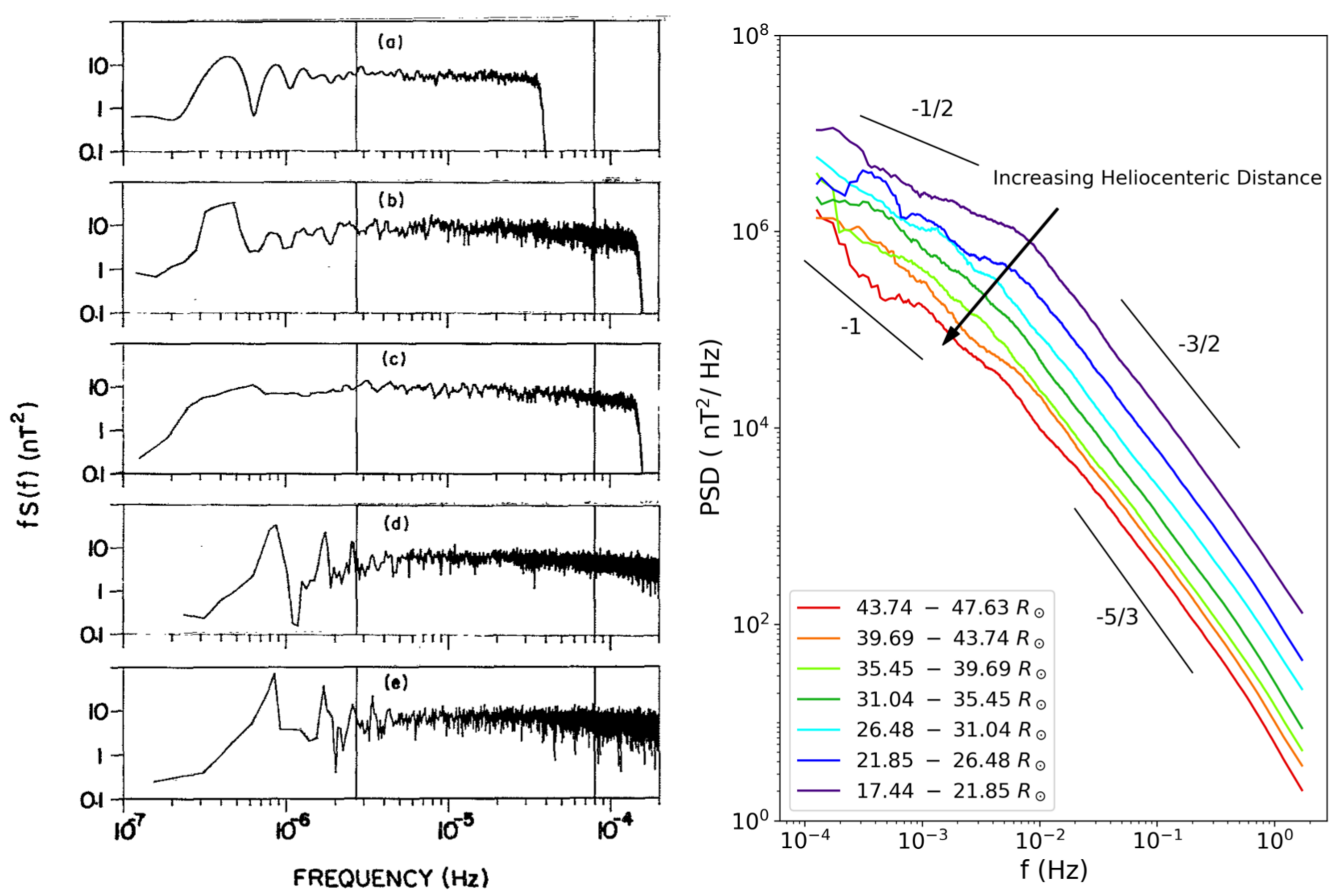
Background

$1/f$ noise, otherwise called “flicker noise”, refers to a region of the power spectrum where the spectral density $S(f)$ inversely varies with frequency f. This spectrum is unique because the integrated power per octave is independent of frequency. Mathematically, the integration $\int S(f)df \sim \int df/f \sim \log f_2/f_1$ relies solely on the ratio between the upper and lower integration bounds, f_2/f_1.

Signals with $1/f$ spectra are found and studied in numerous systems, including semiconductors, music, and human heartbeats. They have also been observed in interplanetary magnetic and density fields since the 1908s[1,2]. Understanding the origin of the interplanetary $1/f$ spectra was counted among the scientific motivations for design of the Parker Solar probe mission[3].

Observations and theoretical issues

$1/f$ spectra typically extend over several orders of magnitude in frequency near 1 au, and transition to steeper $f^{-5/3}$ or $f^{-3/2}$ power laws at higher frequencies, reminiscent of the classical homogeneous, isotropic inertial range turbulence. The “break frequency” is observed to decrease with increasing heliocentric distance.



Left: Matthaeus & Goldstein 1986, Fig. 1. Right: Davis et al. 2023, Fig. 3. In long data records, $1/f$ is seen between 2×10^{-6} and $\sim 10^{-4}$Hz.

The origin of interplanetary $1/f$ noise is a topic of fundamental debate: does this signal arise from local dynamics, such as linear instability or modifications to the principles underlying a Kolmogorov-like cascade[4,5], or from non-local statistical properties possibly near the solar corona, which can be shown mathematically to generate $1/f$ spectra?

Superposition principle for generation of $1/f$ noise

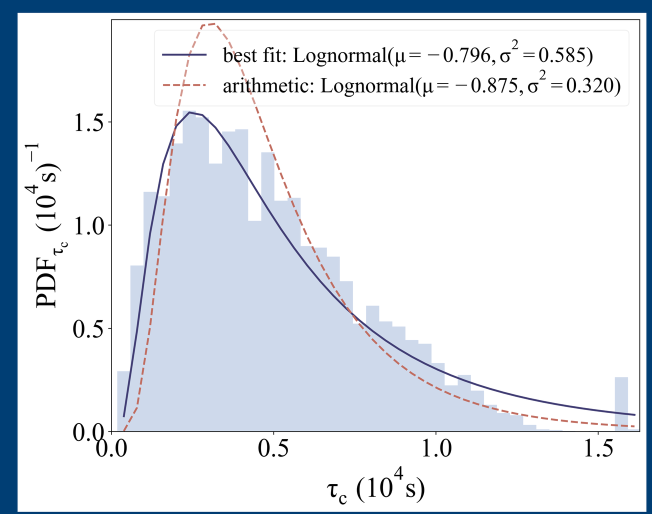
A superposition of purely random processes (those with e^{-t/τ_c} autocorrelations) with a scale-invariant correlation time distribution $\rho(\tau_c)\, d\tau_c \propto d\tau_c/\tau_c$ produces a $1/f$ spectrum[6]:

$$S(\tau_c,\omega) \propto \int_{-\infty}^{\infty} e^{-i\omega t} e^{-t/\tau_c} dt \propto \frac{\tau_c}{1+\omega^2\tau_c^2},$$

$$\overline{S}(\omega) = \int_{\tau_1}^{\tau_2} S(\tau_c,\omega)\rho(\tau_c) d\tau_c \propto \left.\frac{\tan^{-1}(\tau_c\omega)}{\omega}\right|_{\tau_1}^{\tau_2}.$$

Superposing datasets with an arbitrary power-law index less than -1 also gives rise to $1/f$ spectrum.

A lognormal distribution with large variance overlaps with a scale-invariant distribution[7]. Correlation times in the solar wind at 1 au follows a lognormal distribution.



Lognormal distribution naturally arise from multiplicative processes, which may manifest as successive reconnection events or successive foldings in a dynamo[2].

Scan to see the maths.

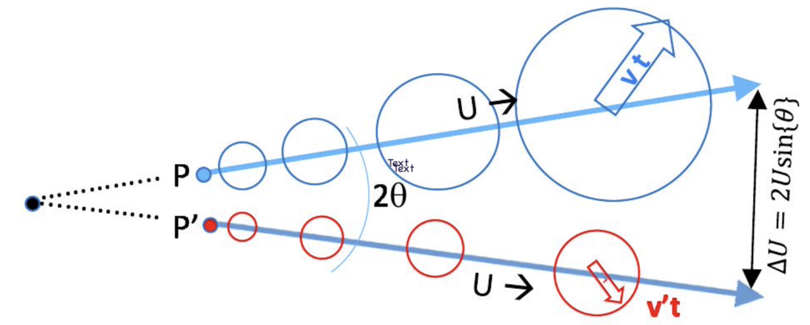
“If you have not found the $1/f$ spectrum, it is because you have not waited long enough[6].”

Coronal and solar origin (superposition)

- The observed $1/f$ spectra may come from the superposition of signals from uncorrelated magnetic reconnection events in the corona, whose respective correlation times collectively exhibit a lognormal-like distribution[8,9,10].
- Inverse cascade systems produce large scale structure and $1/f$ in time domain, which is also seen in dynamo experiments and spherical MHD simulations[11].
- Shell-reduced MHD model with Alfvén waves injected at the base of the chromosphere shows that reflection supports inverse cascade and creates low-frequency $1/f$ spectra[5].
- Azimuthal spectrum of line of sight photospheric magnetic field (from MDI data) shows $1/k$ spectrum that can become $1/f$ after propagation.
- Dynamo simulations indicate origin of $1/f$ in solar dynamo[12,13].

Local origins

- Nonlinear couplings in expanding wind with WKB ordering can create a local self-similar $1/f$ spectrum[4].
- Such mechanism cannot produce $1/f$ over observed range due to causality limitations[14] - MHD cannot communicate over required range in time of convection to 1 au (see Figure).



- Other proposed models for generating $1/f$ might explain observations near 10^{-3} to 10^{-4} Hz, but for essentially the same causality limitation cannot explain observations down to 10^{-6} Hz[15,16].

Observability by PUNCH

The implied sizes of the structures associated with the $1/f$ signal correspond to the range of scales to which PUNCH is sensitive – from a few correlation lengths up to 1 au.
Challenge: How can we connect the images to the $1/f$ structures? (See paper by Pecora+2024)

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