Data Enhanced Modelling of Magnetized Turbulence

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Turbulence: nonlinearity

- Chaotic pattern
- Multi temporal and spatial scales

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V} = -\frac{1}{\rho}\nabla p + \frac{f}{\rho} + \nu\nabla^2\mathbf{V}$$

Ignore RHS

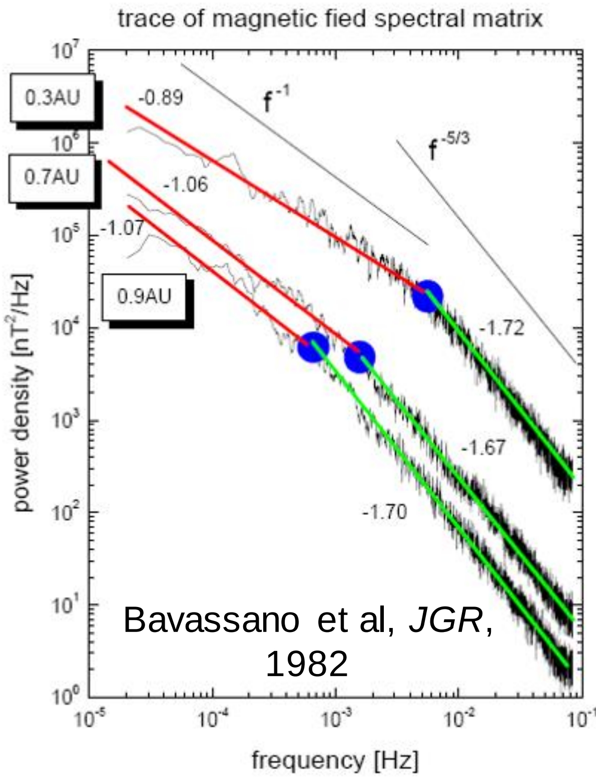
$$\frac{\partial \mathbf{V}}{\partial t} = -\mathbf{V}\cdot\nabla\mathbf{V}$$

$$\mathbf{V}_{t+\delta t} = \mathbf{V}_t - \delta t(\mathbf{V}_t\cdot\nabla\mathbf{V}_t)$$

assume 1D and $V_t \sim \sin(kx)$

$$V_{t+\delta t} = \sin(kx) - \delta t \sin(kx)\cos(kx)$$

$$V_{t+\delta t} = \sin(kx) - \frac{\delta t}{2}\sin(2kx)$$



Bavassano et al, *JGR*, 1982

Turbulence: nonlinearity`

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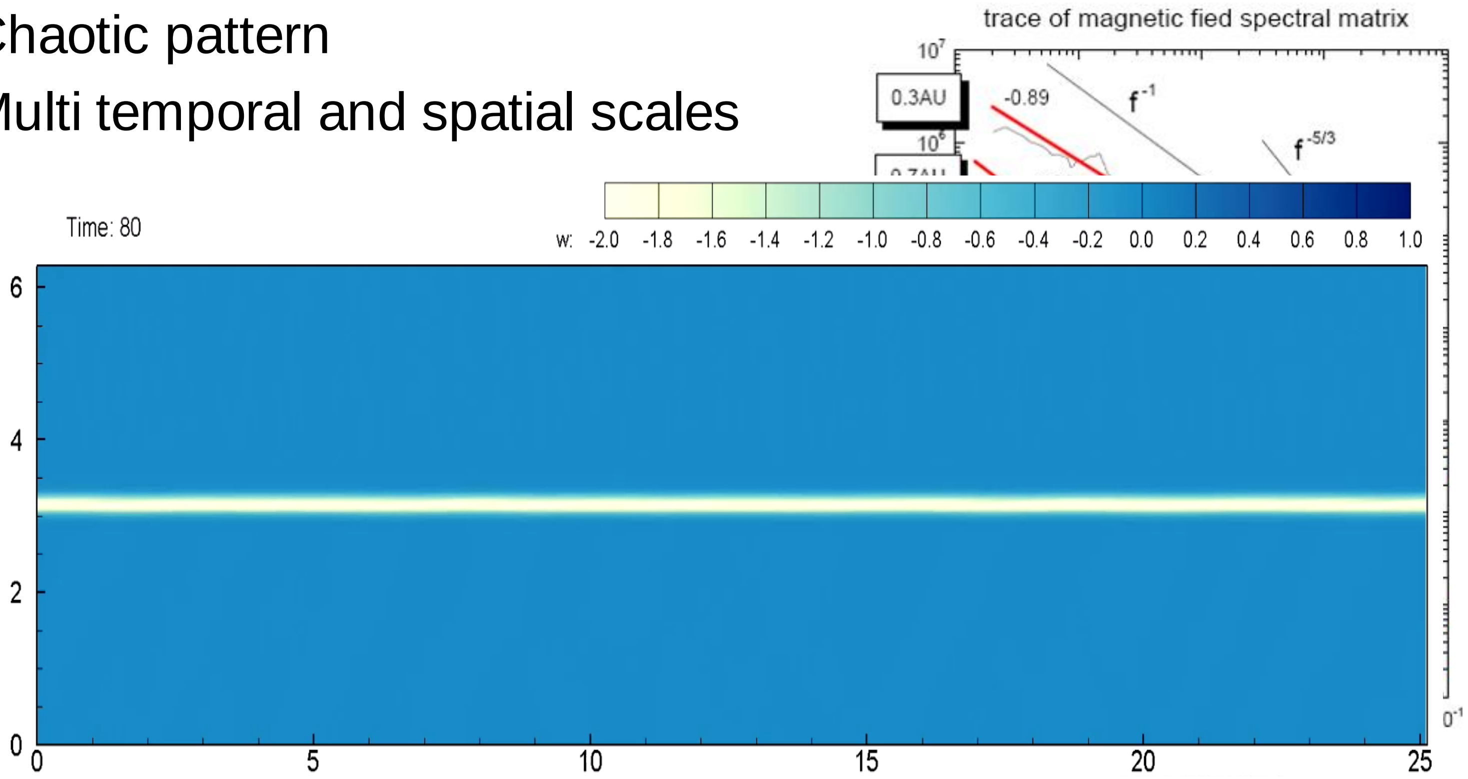


Turbulence Modelling Approaches

- Direct numerical simulation (DNS)

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \boldsymbol{u}$$

- Large-eddy simulation (LES)

$$\partial_t \widetilde{\boldsymbol{u}} + \widetilde{\boldsymbol{u}} \cdot \nabla \widetilde{\boldsymbol{u}} = -\frac{\nabla \tilde{p}}{\rho} + \nu \nabla^2 \widetilde{\boldsymbol{u}} - \boldsymbol{\nabla} \cdot \boldsymbol{\tau}^l$$

$$\tau_{ij}^l = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$$

- Reynolds-averaged Navier-Stokes model (RANS)

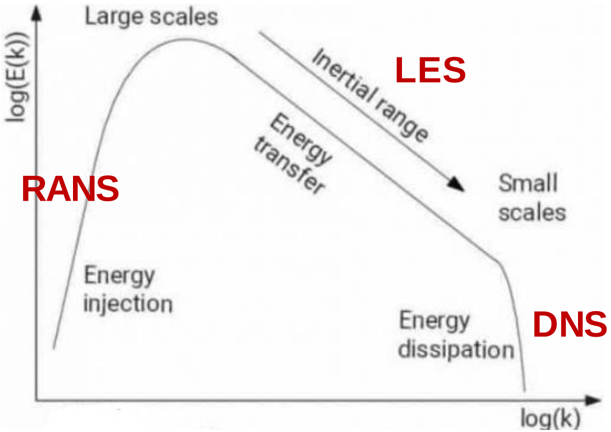
$$\boldsymbol{u} = \langle \boldsymbol{u} \rangle + \boldsymbol{u}'$$

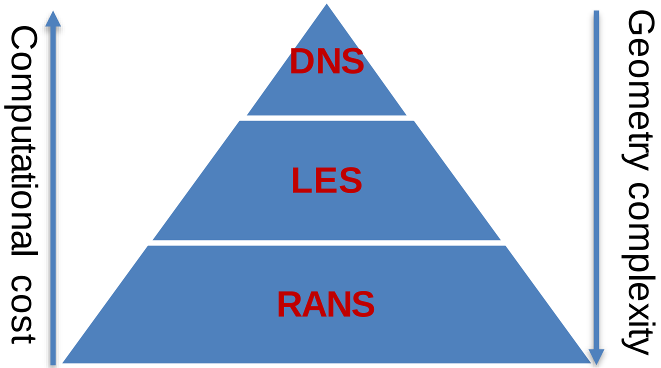
$$\partial_t \langle \boldsymbol{u} \rangle + \langle \boldsymbol{u} \rangle \cdot \nabla \langle \boldsymbol{u} \rangle = -\frac{\nabla \langle p \rangle}{\rho} + \nu \nabla^2 \langle \boldsymbol{u} \rangle - \boldsymbol{\nabla} \cdot \boldsymbol{\tau}$$

$$\tau_{ij} = \langle u_i u_j \rangle$$

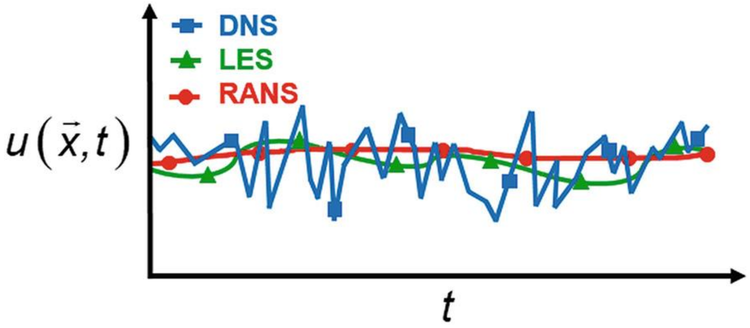
Turbulence Modelling Approaches

Model	Level of description	Completeness	Cost and ease of use
DNS	Kolmogorov scale	Complete	High $\propto Re^{2.64}$ Choi and Moin, 2012
LES	Inertial range	Incomplete	Mediate $\propto Re^{1.85}$
RANS	Large-scale mean flow	Incomplete	Low

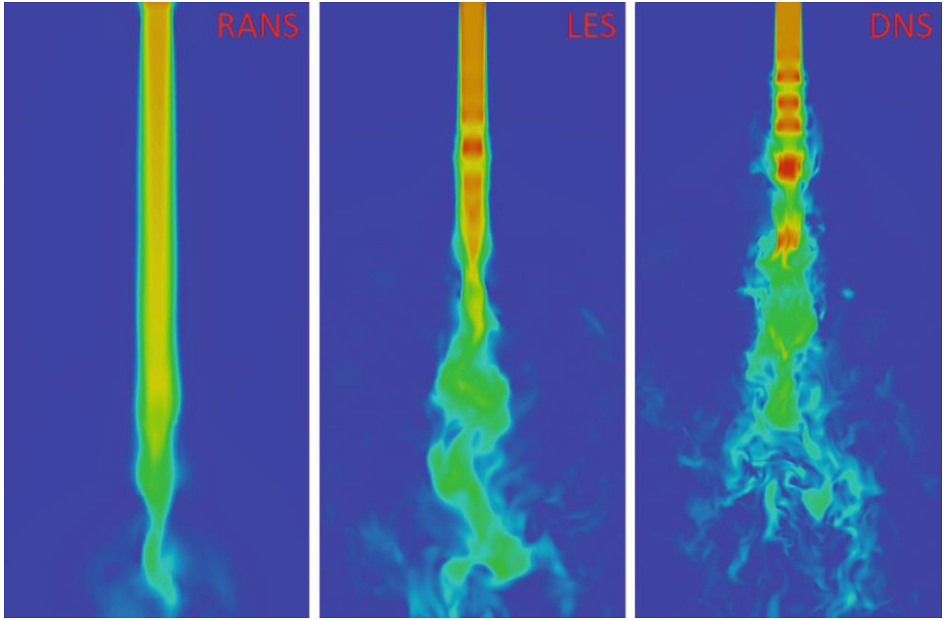




Turbulence Modelling Approaches



Rodriguez 2019



Velocity distribution for a turbulent jet

LES

$$\partial_t \tilde{\boldsymbol{u}} + \tilde{\boldsymbol{u}} \cdot \nabla \tilde{\boldsymbol{u}} = -\frac{\nabla \tilde{p}}{\rho} + \nu \nabla^2 \tilde{\boldsymbol{u}} - \boldsymbol{\nabla} \cdot \boldsymbol{\tau}^l$$

$$\tau_{ij}^l = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$$

Scale invariance:
certain features of the flow remain the same in different scales of motion

Big whorls have little whorls, which feed on their velocity, and little whorls have lesser whorls, and so on to viscosity (in the molecular sense).
Richardson 1922

A partial list of models:
Standard Smagorinsky model (Smagorinsky 1963)

Similarity model (Bardina 1980)

Dynamic model (Germano et al 1991)

……

LES

Diagnostic variable	M1	M2	M3	M4	M5	M6
Total kinetic energy $\left[\int \frac{1}{2} \rho \tilde{u}_i \tilde{u}_i \, d^3\mathbf{x}\right]$	−	+	−	0	+ +	+
SGS dissipation $\left[\int -(\rho \tau_{ij}^{\Delta} \tilde{S}_{ij}) \, d^3\mathbf{x}\right]$	−	+	−	+ +	+	+
Backscatter $\left[\int \min(-\rho \tau_{ij}^{\Delta} \tilde{S}_{ij}, 0) \, d^3\mathbf{x}\right]$	−	0	−	−	0	0
Stress magnitude [L_2 norm of τ_{12}^{Δ}]	−	+	0	−	+	+
Energy spectrum $E(k_1)$	−	−	−	+	+ +	0
Vorticity in a plane [$\tilde{\omega}_3(x_1, x_2)$]	−	−	−	+	+ +	+
Maximum vorticity	−	−	−	+	+	0
Momentum thickness	−	+	−	−	+	+ +
$\langle \tilde{u}_1' \tilde{u}_1' \rangle$	−	+	−	+	+	+ +
$-\langle \tilde{u}_1' \tilde{u}_2' \rangle$	−	−	−	0	+ +	+

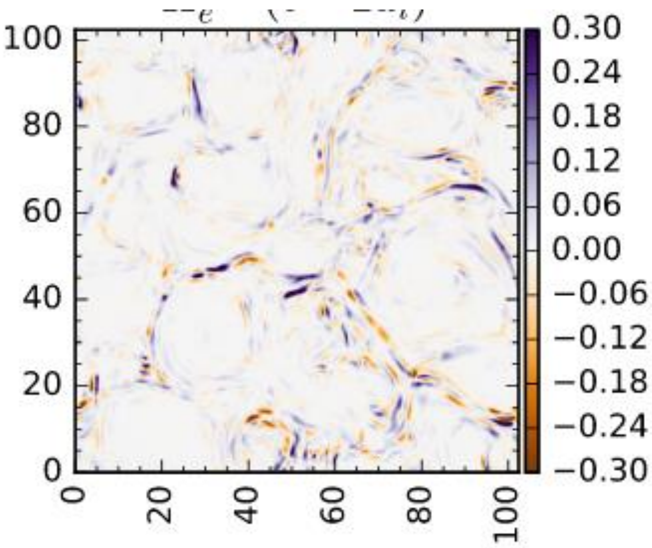
[a]The symbols −, 0, and + refer to bad, reasonable, and good results, respectively. + + is better than +. M1 = constant coefficient Smagorinsky model; M2 = pure similarity model with $C_{\mathrm{sim}} = 1$ and $\gamma = 1$ without eddy viscosity; M3 = nonlinear model with $C_{\mathrm{nl}} = 1/12$, without eddy viscosity but with a clipping procedure to avoid backscatter; M4 = dynamic Smagorinsky; M5 = dynamic mixed model with $\gamma = 1$ and $C_{\mathrm{sim}} = 1$; M6 = dynamic mixed nonlinear model, with $C_{\mathrm{nl}} = 1/12$. Dynamic models use averaging over the homogeneous direction ($x_1 - x_3$ planes).

Vreman 1997

Smagorinsky Model

$$\tau_{ij}^{l} - \frac{1}{3}\tau_{kk}^{l}\delta_{ij} = -2\nu_T \tilde{S}_{ij}$$
$$\tilde{S}_{ij} = \frac{1}{2}\left(\partial_j \tilde{u}_i + \partial_i \tilde{u}_j\right)$$
$$\nu_T = (C_s l)^2 |\tilde{S}|$$

Local SGS dissipation could be negative, while the smagorinsky model is always purely dissipative, $-\tau_{ij}^{l}\tilde{S}_{ij} \geq 0$.



Yang et al 2017

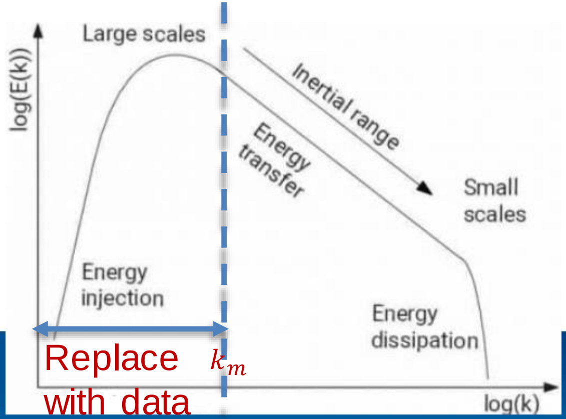
Data Assimilation

- Four-dimensional variational (4DVAR) method (Lions 1971)

$$J = \frac{1}{2}\int_0^T \langle \mathcal{F}\boldsymbol{u} - \mathcal{F}\boldsymbol{v}, \mathcal{F}\boldsymbol{u} - \mathcal{F}\boldsymbol{v}\rangle \, \mathrm{d}t$$

$$-\partial_t \boldsymbol{u} - \boldsymbol{u}\cdot\nabla\boldsymbol{u} - \nabla p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f} = 0, \quad \nabla\cdot\boldsymbol{u} = 0, \quad \boldsymbol{u}(\boldsymbol{x},0) = \boldsymbol{\varphi}(\boldsymbol{x})$$

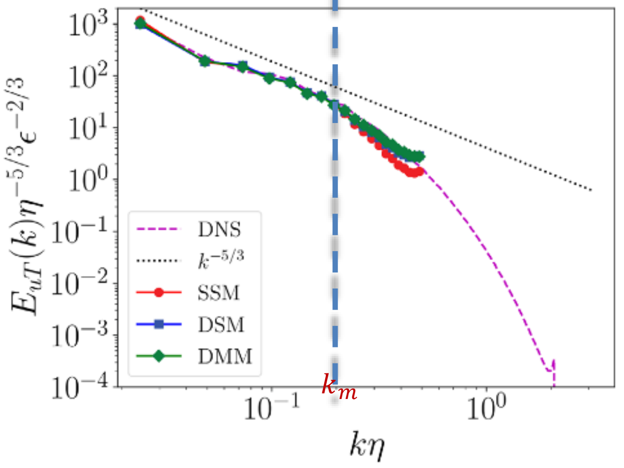
- Ensemble Kalman filter (EnKF) method (Kalman 1960)
- Direct substitution (DS) method (Yoshida et al 2005)

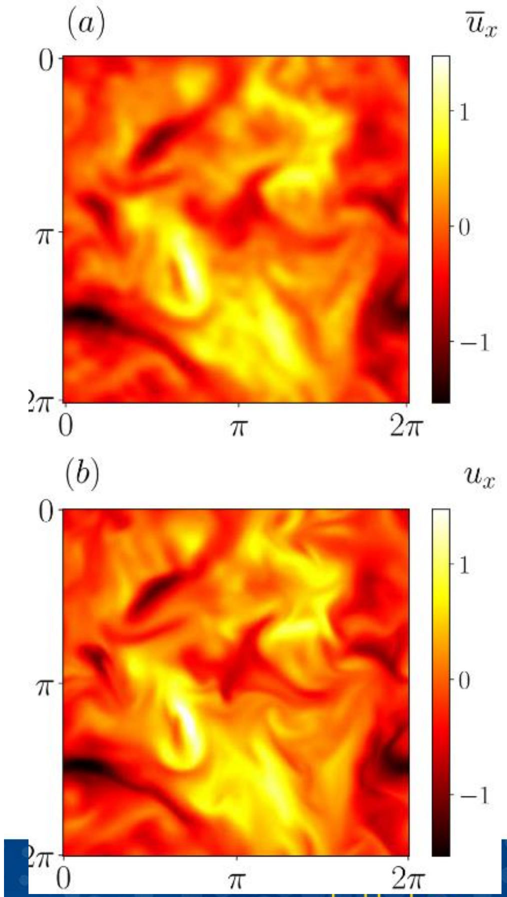


Synchronization of LES

- When the amount of assimilated data exceeds a threshold given by a threshold wavenumber k_m, large eddy simulations is synchronized with the direct numerical simulations in phase.
- DA significantly improve the prediction of the instantaneous point-wise distribution of the velocity field.
- DA does not significantly alter the statistics produced by the SGS models.

Li et al 2022



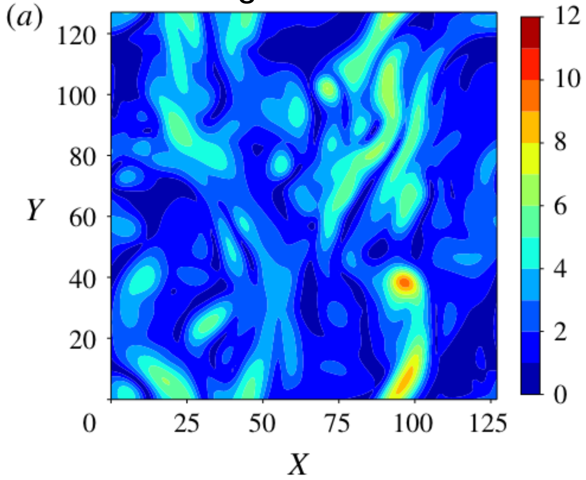


Small-Scale Reconstruction of Turbulence

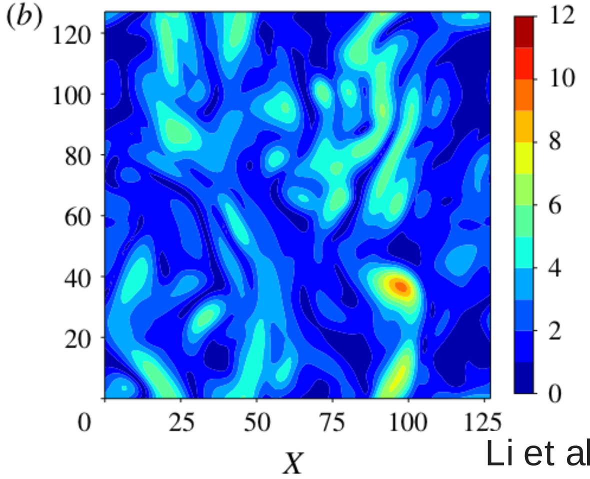
- Reconstruction is successful when the resolution of the measurement data, is of the order of the threshold value $k_c = 0.2\eta_K^{-1}$.

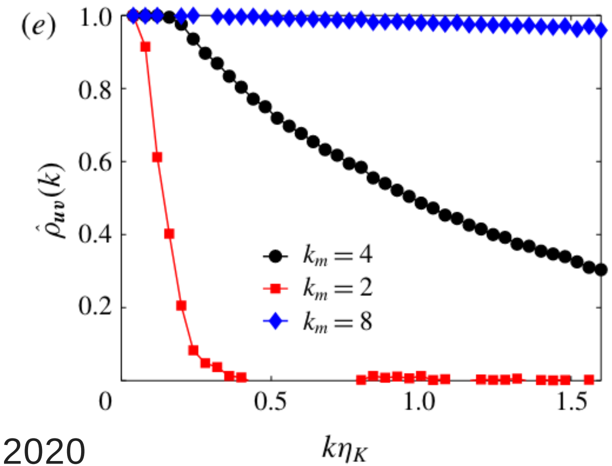
- Satisfactory reconstruction of the scales two or more octaves smaller is possible if data at large scales are available for at least one large eddy turnover time.

Target field



Reconstructed field





Li et al 2020

Summary

- Resolving all scales in turbulence is challenging. LES is computationally feasible and captures some turbulent features.

- Data assimilation (DA) can be used to improve the prediction of LES and reconstruct small scales.

- Given that PUNCH will admit sufficiently high spatial resolution to probe scales of turbulence within the upper end of inertial range, DA can be used to synthesize available PUNCH data at large scales and numerical simulations to improve the prediction capability of numerical models, in particular, at small scales.